Online Appendix A

Proof of the Effects of Ignoring Clustering

This appendix discusses about how to estimate the differences in parameters, including standardized factor loadings and factor correlations. First, model implied correlation matrices for both levels are created by

$$\boldsymbol{\Sigma}_B^* = \boldsymbol{\Lambda}_B^* \boldsymbol{\Psi}_B^* \boldsymbol{\Lambda}_B^{*T} + \boldsymbol{\Theta}_B^* \quad \text{(A1)}$$

$$\boldsymbol{\Sigma}_W^* = \boldsymbol{\Lambda}_W^* \boldsymbol{\Psi}_W^* \boldsymbol{\Lambda}_W^{*T} + \boldsymbol{\Theta}_W^* \quad \text{(A2)}$$

where $\boldsymbol{\Sigma}_B^*$ and $\boldsymbol{\Sigma}_W^*$ are model-implied correlation matrices for the macro and micro levels, respectively. $\boldsymbol{\Lambda}_B^*$ and $\boldsymbol{\Lambda}_W^*$ are standardized factor loading matrices for the macro and micro levels, respectively. $\boldsymbol{\Psi}_B^*$ and $\boldsymbol{\Psi}_W^*$ are factor correlation matrices for the macro and micro levels, respectively. $\boldsymbol{\Theta}_B^*$ and $\boldsymbol{\Theta}_W^*$ are (usually diagonal) unique variance matrices for the macro and micro levels, respectively. The observed covariance matrix in each level can be calculated from the observed correlation from each level by

$$\boldsymbol{\Sigma}_B = \mathbf{D}_{BY}^{1/2} \boldsymbol{\Sigma}_B^* \mathbf{D}_{BY}^{1/2} \quad \text{(A3)}$$

$$\boldsymbol{\Sigma}_W = \mathbf{D}_{WY}^{1/2} \boldsymbol{\Sigma}_W^* \mathbf{D}_{WY}^{1/2} \quad \text{(A4)}$$

where $\boldsymbol{\Sigma}_B$ and $\boldsymbol{\Sigma}_W$ are the observed covariance matrices for the macro and micro levels, respectively, $\mathbf{D}_{BY}$ and $\mathbf{D}_{WY}$ are the diagonal matrices containing the variances of observed variables. In other words, $\mathbf{D}_{BY} = diag(\boldsymbol{\Sigma}_B)$ and $\mathbf{D}_{WY} = diag(\boldsymbol{\Sigma}_W)$, where $diag$ is the function to extract only diagonal elements. Let $\mathbf{P}$ be a $p \times p$ diagonal matrix containing ICCs of all variables and $\mathbf{D}_Y$ be a $p \times p$ diagonal matrix containing the total observed variance of each variable. Then,

$$\mathbf{D}_{BY} = \mathbf{P}\mathbf{D}_Y \quad \text{(A5)}$$

$$\mathbf{D}_{WY} = (\mathbf{I} - \mathbf{P})\mathbf{D}_Y \quad \text{(A6)}$$

where $\mathbf{I}$ is a $p \times p$ identity matrix. The observed covariance matrices from the disaggregation and aggregation methods, denoted by $\mathbf{\Sigma}_D$ and $\mathbf{\Sigma}_A$, respectively, will be

$$\mathbf{\Sigma}_D = \mathbf{\Sigma}_B + \mathbf{\Sigma}_W = (\mathbf{P}\mathbf{D}_Y)^{1/2}\mathbf{\Sigma}_B^*(\mathbf{P}\mathbf{D}_Y)^{1/2} + \left((\mathbf{I}-\mathbf{P})\mathbf{D}_Y\right)^{1/2}\mathbf{\Sigma}_W^*\left((\mathbf{I}-\mathbf{P})\mathbf{D}_Y\right)^{1/2} \quad \text{(A7)}$$

$$\begin{aligned}\mathbf{\Sigma}_A &= \mathbf{\Sigma}_B + n^{-1}\mathbf{\Sigma}_W \\ &= (\mathbf{P}\mathbf{D}_Y)^{1/2}\mathbf{\Sigma}_B^*(\mathbf{P}\mathbf{D}_Y)^{1/2} \\ &\quad + n^{-1}\left((\mathbf{I}-\mathbf{P})\mathbf{D}_Y\right)^{1/2}\mathbf{\Sigma}_W^*\left((\mathbf{I}-\mathbf{P})\mathbf{D}_Y\right)^{1/2}\end{aligned} \quad \text{(A8)}$$

where n is cluster size, assuming that all clusters have equal size. The disaggregated and aggregated observed variance, denoted as $\mathbf{D}_{DY}$ and $\mathbf{D}_{AY}$, can be calculated by

$$\mathbf{D}_{DY} = diag(\mathbf{\Sigma}_D) = diag(\mathbf{\Sigma}_B) + diag(\mathbf{\Sigma}_W) = \mathbf{D}_{BY} + \mathbf{D}_{WY} = \mathbf{D}_Y \quad \text{(A9)}$$

$$\begin{aligned}\mathbf{D}_{AY} &= diag(\mathbf{\Sigma}_A) = diag(\mathbf{\Sigma}_B) + n^{-1}diag(\mathbf{\Sigma}_W) = \mathbf{D}_{BY} + n^{-1}\mathbf{D}_{WY} \\ &= \left(\mathbf{P} + n^{-1}(\mathbf{I}-\mathbf{P})\right)\mathbf{D}_Y\end{aligned} \quad \text{(A10)}$$

Then, $\mathbf{\Sigma}_D$ and $\mathbf{\Sigma}_A$ are used to find parameters in single-level CFA model. Maximum likelihood can be used in parameter estimation. If researchers need SE and fit indices, sample size is required. Another method is to use the instrumental variable method (Hägglund, 1982). This method can be used to find a closed form equation for estimating the differences in parameter estimations. Therefore, we use this method to show that standardized coefficients are not deviated in some special cases with a large ICC, in contrast to Julian's (2001) results showing that ignoring clustering always leads to differences when ICC is large. Readers may be unfamiliar with the instrumental variable method. Therefore, we will explain this instrumental variable method first and then derive the closed form formula for the difference in standardized parameter estimation.

Instrumental Variable Method for Factor Analysis

Hägglund (1982) described how to use the instrumental variable method to estimate parameters in both exploratory and confirmatory factor analysis. We use this method to estimate parameters in confirmatory factor analysis. This method can be used to obtain parameter estimates in two steps. First, factor loadings are estimated using the instrumental variable technique. Then, the obtained factor loading will be used in the estimation of unique factor variances and common factor covariances.

For factor loading estimation, factor loadings of each factor will be analyzed separately. Within each factor, indicators are divided to three sets: one marker variable, one target variable, and the rest that are used as instrumental variables. The marker variable is the variable that is used for scaling, which should well represent a desired factor. The factor loadings of the marker variable are fixed to 1 for the desired factor and to 0 for all other factors. The target variable is the variable for which we wish to estimate a factor loading. To estimate all factor loadings within a factor, all variables in a factor will take turns to be the target variable until all factor loadings are estimated, except marker variables.

By definition, a factor loading is a regression coefficient for predicting an indicator by a factor. Because the marker variable represents a factor (the loading equals 1), we can predict the target variable with the marker variable to obtain an approximate factor loading. However, in this case, measurement error in this equation (the regression residual) will be related to the factor. This relationship violates a fundamental assumption of regression analysis—that predictors are independent of regression errors, leading to an inaccurate estimation of the factor loading. Therefore, the instrumental variables (the remaining indicators) are used to partial out the portion of variance in the marker variable that potentially correlates with measurement error (or the regression residual). That is, the marker variable is regressed on the instrument variables and the residuals are saved. Next, the residuals will be used to predict the target variable. The factor loading from this prediction will be accurately estimated.

Assume that we are interested only in the factor structure in which each indicator loads on one factor only (factor loadings with other factors all equal 0). Let Variable Sets 1, 2, and 3 be the marker variable, the target variable, and the instrumental variables, respectively. The number of indicators in Sets 1 and 2 is 1. The number of indicators in Set 3 is at least 1. To calculate the factor loading on each factor, the indicator covariance matrix that is used for calculation is the covariance matrix that involves the indicators that have nonzero loadings on the factor, which will be referred to as the reduced observed covariance matrix, $\mathbf{\Sigma}_R$. The reduced covariance matrix will be affected by only one factor, which has a scalar factor variance. That is, $\mathbf{\Psi}_B = \psi_B$ and $\mathbf{\Psi}_W = \psi_W$. In the standardized metric, the factor variance will be 1: $\mathbf{\Psi}_B^* = \mathbf{\Psi}_W^* = 1$. Therefore, the reduced covariance matrix will be

$$\mathbf{\Sigma}_R = \mathbf{D}_{RY}^{1/2}\mathbf{\Sigma}_R^*\mathbf{D}_{RY}^{1/2} = \mathbf{D}_{RY}^{1/2}(\mathbf{\Lambda}_R^*\mathbf{\Lambda}_R^{*T} + \mathbf{\Theta}_R^*)\mathbf{D}_{RY}^{1/2} \tag{A11}$$

where $\mathbf{D}_{RY}$ is the diagonal matrix of variable variances. The reduced observed covariance matrix can be partitioned based on the set of variables as

$$\mathbf{\Sigma}_R = \begin{bmatrix} \mathbf{\Sigma_{11}} & \mathbf{\Sigma_{12}} & \mathbf{\Sigma_{13}} \\ \mathbf{\Sigma_{21}} & \mathbf{\Sigma_{22}} & \mathbf{\Sigma_{23}} \\ \mathbf{\Sigma_{31}} & \mathbf{\Sigma_{32}} & \mathbf{\Sigma_{33}} \end{bmatrix} \tag{A12}$$

Next, we will map the blocks of covariances with the blocks of factor loadings and uniqueness:

$$\mathbf{\Lambda}_R^* = \begin{bmatrix} \mathbf{\Lambda}_1^* \\ \mathbf{\Lambda}_2^* \\ \mathbf{\Lambda}_3^* \end{bmatrix} \tag{A13}$$

$$\mathbf{\Theta}_R^* = \begin{bmatrix} \mathbf{\Theta}_1^* & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Theta}_2^* & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Theta}_3^* \end{bmatrix} \tag{A14}$$

$$\mathbf{D}_{RY}^{1/2} = \begin{bmatrix} \mathbf{D}_{R1}^{1/2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{R2}^{1/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{D}_{R3}^{1/2} \end{bmatrix} \tag{A15}$$

Then, from Equations A12-A15,

$$\begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} & \boldsymbol{\Sigma}_{13} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} & \boldsymbol{\Sigma}_{23} \\ \boldsymbol{\Sigma}_{31} & \boldsymbol{\Sigma}_{32} & \boldsymbol{\Sigma}_{33} \end{bmatrix} = \mathbf{D}_R^{1/2} \begin{bmatrix} \boldsymbol{\Lambda}_1^*\boldsymbol{\Lambda}_1^{*T} + \boldsymbol{\Theta}_1^* & \boldsymbol{\Lambda}_1^*\boldsymbol{\Lambda}_2^{*T} & \boldsymbol{\Lambda}_1^*\boldsymbol{\Lambda}_3^{*T} \\ \boldsymbol{\Lambda}_2^*\boldsymbol{\Lambda}_1^{*T} & \boldsymbol{\Lambda}_2^*\boldsymbol{\Lambda}_2^{*T} + \boldsymbol{\Theta}_2^* & \boldsymbol{\Lambda}_2^*\boldsymbol{\Lambda}_3^{*T} \\ \boldsymbol{\Lambda}_3^*\boldsymbol{\Lambda}_1^{*T} & \boldsymbol{\Lambda}_3^*\boldsymbol{\Lambda}_2^{*T} & \boldsymbol{\Lambda}_3^*\boldsymbol{\Lambda}_3^{*T} + \boldsymbol{\Theta}_3^* \end{bmatrix} \mathbf{D}_R^{1/2} \tag{A16}$$

$$= \begin{bmatrix} \mathbf{D}_{R1}^{1/2}(\boldsymbol{\Lambda}_1^*\boldsymbol{\Lambda}_1^{*T} + \boldsymbol{\Theta}_1^*)\mathbf{D}_{R1}^{1/2} & \mathbf{D}_{R1}^{1/2}(\boldsymbol{\Lambda}_1^*\boldsymbol{\Lambda}_2^{*T})\mathbf{D}_{R2}^{1/2} & \mathbf{D}_{R1}^{1/2}(\boldsymbol{\Lambda}_1^*\boldsymbol{\Lambda}_3^{*T})\mathbf{D}_{R3}^{1/2} \\ \mathbf{D}_{R2}^{1/2}(\boldsymbol{\Lambda}_2^*\boldsymbol{\Lambda}_1^{*T})\mathbf{D}_{R1}^{1/2} & \mathbf{D}_{R2}^{1/2}(\boldsymbol{\Lambda}_2^*\boldsymbol{\Lambda}_2^{*T} + \boldsymbol{\Theta}_2^*)\mathbf{D}_{R2}^{1/2} & \mathbf{D}_{R2}^{1/2}(\boldsymbol{\Lambda}_2^*\boldsymbol{\Lambda}_3^{*T})\mathbf{D}_{R3}^{1/2} \\ \mathbf{D}_{R3}^{1/2}(\boldsymbol{\Lambda}_3^*\boldsymbol{\Lambda}_1^{*T})\mathbf{D}_{R1}^{1/2} & \mathbf{D}_{R3}^{1/2}(\boldsymbol{\Lambda}_3^*\boldsymbol{\Lambda}_2^{*T})\mathbf{D}_{R2}^{1/2} & \mathbf{D}_{R3}^{1/2}(\boldsymbol{\Lambda}_3^*\boldsymbol{\Lambda}_3^{*T} + \boldsymbol{\Theta}_3^*)\mathbf{D}_{R3}^{1/2} \end{bmatrix}$$

The factor loading of the target variable, $\boldsymbol{\lambda}_2$, is calculated by

$$\boldsymbol{\lambda}_2 = (\boldsymbol{\Sigma}_{13}\boldsymbol{\Sigma}_{31})^{-1}\boldsymbol{\Sigma}_{13}\boldsymbol{\Sigma}_{32} \tag{A17}$$

Note that $\boldsymbol{\lambda}_2$ will have 1 row and 1 column, yielding a scalar. Equation A17 can be expressed in terms of standardized factor loadings as

$$\boldsymbol{\lambda}_2 = \left(\mathbf{D}_{R1}^{1/2}(\boldsymbol{\Lambda}_1^*\boldsymbol{\Lambda}_3^{*T})\mathbf{D}_{R3}^{1/2}\mathbf{D}_{R3}^{1/2}(\boldsymbol{\Lambda}_3^*\boldsymbol{\Lambda}_1^{*T})\mathbf{D}_{R1}^{1/2}\right)^{-1}\mathbf{D}_{R1}^{1/2}(\boldsymbol{\Lambda}_1^*\boldsymbol{\Lambda}_3^{*T})\mathbf{D}_{R3}^{1/2}\mathbf{D}_{R3}^{1/2}(\boldsymbol{\Lambda}_3^*\boldsymbol{\Lambda}_2^{*T})\mathbf{D}_{R2}^{1/2} \tag{A18}$$

As described above, all variables can act in turn as the target variable. Therefore, all factor loadings in the model can be solved for except the marker variables' loadings, which are fixed to 1. Let $\boldsymbol{\Lambda}$ be a factor loading matrix from all indicators of all factors. The unique factor covariance matrix, $\boldsymbol{\Theta}$, which is a diagonal matrix of unique variances, is calculated by

$$\text{diag}(\boldsymbol{\Theta}) = \mathbf{E}^{-1}\mathbf{g} \tag{A19}$$

where

$$\mathbf{E} = \mathbf{I} - \mathbf{K} * \mathbf{K} \tag{A20}$$

$$\mathbf{g} = \text{diag}(\boldsymbol{\Sigma} - \mathbf{K}\boldsymbol{\Sigma}\mathbf{K}) \tag{A21}$$

$$\mathbf{K} = \boldsymbol{\Lambda}(\boldsymbol{\Lambda}^T\boldsymbol{\Lambda})^{-1}\boldsymbol{\Lambda}^T \tag{A22}$$

where * represents elementwise multiplication, and $\mathbf{\Sigma}$ is a covariance matrix among indicators. Next, the factor covariance matrix, $\mathbf{\Psi}$, is estimated by

$$\mathbf{\Psi} = (\mathbf{\Lambda}^T\mathbf{\Lambda})^{-\mathbf{1}}\mathbf{\Lambda}^T(\mathbf{\Sigma} - \mathbf{\Theta})\mathbf{\Lambda}\,(\mathbf{\Lambda}^T\mathbf{\Lambda})^{-\mathbf{1}} \tag{A23}$$

To find standardized parameter estimates, let $\mathbf{D}_\eta$ be a an *m* × *m* diagonal matrix with factor variances as diagonal elements and $\mathbf{D}_Y$ be a *p* × *p* diagonal matrix with indicator variances as diagonal elements:

$$\mathbf{D}_\eta = \text{diag}(\mathbf{\Psi}) \tag{A24}$$

$$\mathbf{D}_Y = \text{diag}(\mathbf{\Sigma}) \tag{A25}$$

Letting starred matrices be standardized matrices, standardized factor loading and factor correlations are given by:

$$\mathbf{\Lambda}^* = \mathbf{D}_Y^{-1/2}\mathbf{\Lambda}\mathbf{D}_\eta^{1/2} \tag{A26}$$

$$\mathbf{\Psi}^* = \mathbf{D}_\eta^{-1/2}\mathbf{\Psi}\mathbf{D}_\eta^{-1/2} \tag{A27}$$

The objective of this appendix is to derive the differences in parameter estimates from CFA analyses using disaggregation and aggregation. The disaggregation parameter estimates are compared with the parameter estimates for the micro level. Thus, the difference in standardized factor loadings after disaggregation is $\mathbf{\Lambda}_D^* - \mathbf{\Lambda}_W^*$ and the difference in factor correlations after disaggregation is $\mathbf{\Psi}_D^* - \mathbf{\Psi}_W^*$. Aggregation is usually used to obtain parameter estimates for the macro level. After aggregation, the difference in standardized factor loadings is $\mathbf{\Lambda}_A^* - \mathbf{\Lambda}_B^*$ and the difference in factor correlations is $\mathbf{\Psi}_A^* - \mathbf{\Psi}_B^*$.

In the following we derive the differences in standardized loadings and factor correlations after both aggregation and disaggregation in three cases. First, we will show that the standardized factor loading and factor correlation in the aggregation and disaggregation approaches are not deviated when ICCs are equal across indicators and all standardized parameters from both levels are equal. Next, we will show the closed form formula for the aggregated and disaggregated standardized parameters when ICCs are equal across items, standardized factor loadings are equal across levels, and factor correlations are different across levels. Finally, we will show the closed form formula for the aggregated and disaggregated standardized parameters when ICCs are equal across items, factor correlations are equal across levels, and the standardized factor loadings are proportional across levels. We will show that the closed form formulas of the second and third scenarios are the same.

Case 1: Standardized Factor Loading and Factor Correlation Invariance

ICCs are equal across all indicators: $\mathbf{P} = \rho\mathbf{I}$. Factor structures are the same in both levels. Standardized parameters are equal across levels: $\boldsymbol{\Lambda}_B^* = \boldsymbol{\Lambda}_W^*$ and $\boldsymbol{\Psi}_B^* = \boldsymbol{\Psi}_W^*$.

From Equation A5, A6, A7, and A8, the disaggregated and aggregated observed covariances are

$$\boldsymbol{\Sigma}_D = \frac{1}{1-\rho}\boldsymbol{\Sigma}_W \tag{A28}$$

$$\boldsymbol{\Sigma}_A = \left(\frac{\rho(n-1)+1}{\rho n}\right)\boldsymbol{\Sigma}_B \tag{A29}$$

To find the disaggregated and aggregated factor loadings in terms of the micro and macro factor loadings, respectively, by using Equation A18, two intermediate steps are required. First, we need the disaggregated and aggregated observed variance, denoted $\mathbf{D}_{DY}$ and $\mathbf{D}_{AY}$, in terms of the micro and macro observed variances. From Equations A28 and A29, it can be shown that

$$\mathbf{D}_{DY} = \mathbf{D}_Y = \left(\frac{1}{1-\rho}\right)\mathbf{D}_{WY} \tag{A30}$$

$$\mathbf{D}_{AY} = \left(\frac{\rho(n-1)+1}{n}\right)\mathbf{D}_Y = \left(\frac{\rho(n-1)+1}{\rho n}\right)\mathbf{D}_{BY} \tag{A31}$$

Second, let k and l be 1, 2, or 3 and $k \neq l$. We need to find $\mathbf{\Lambda}^*_{Dk}\mathbf{\Lambda}^{*T}_{Dl}$ and $\mathbf{\Lambda}^*_{Ak}\mathbf{\Lambda}^{*T}_{Al}$ in terms of $\mathbf{\Lambda}^*_{Wk}\mathbf{\Lambda}^{*T}_{Wl}$ and $\mathbf{\Lambda}^*_{Bk}\mathbf{\Lambda}^{*T}_{Bl}$, respectively. From Equation A3, A4, A7, and A16, it can be shown that

$$\begin{aligned}\mathbf{\Sigma}_{Dkl} = \mathbf{\Sigma}_{Bkl} + \mathbf{\Sigma}_{Wkl} &= \mathbf{D}^{1/2}_{Bk}(\mathbf{\Lambda}^*_{Bk}\mathbf{\Lambda}^{*T}_{Bl})\mathbf{D}^{1/2}_{Bl} + \mathbf{D}^{1/2}_{Wk}(\mathbf{\Lambda}^*_{Wk}\mathbf{\Lambda}^{*T}_{Wl})\mathbf{D}^{1/2}_{Wl} \\ &= \rho^{1/2}\mathbf{D}^{1/2}_{k}(\mathbf{\Lambda}^*_{Wk}\mathbf{\Lambda}^{*T}_{Wl})\rho^{1/2}\mathbf{D}^{1/2}_{l} \\ &+ (1-\rho)^{1/2}\mathbf{D}^{1/2}_{k}(\mathbf{\Lambda}^*_{Wk}\mathbf{\Lambda}^{*T}_{Wl})(1-\rho)^{1/2}\mathbf{D}^{1/2}_{l} \\ &= \mathbf{D}^{1/2}_{k}(\mathbf{\Lambda}^*_{Wk}\mathbf{\Lambda}^{*T}_{Wl})\mathbf{D}^{1/2}_{l}\end{aligned} \tag{A32}$$

From Equations A3 and A30, it can also be shown that

$$\mathbf{\Sigma}_{Dkl} = \mathbf{D}^{1/2}_{Dk}(\mathbf{\Lambda}^*_{Dk}\mathbf{\Lambda}^{*T}_{Dl})\mathbf{D}^{1/2}_{El} = \mathbf{D}^{1/2}_{k}(\mathbf{\Lambda}^*_{Dk}\mathbf{\Lambda}^{*T}_{Dl})\mathbf{D}^{1/2}_{l} \tag{A33}$$

From Equations A32 and A33 and $\mathbf{D}^{1/2}_{k}$ and $\mathbf{D}^{1/2}_{l}$ having inverses, it follows that

$$\mathbf{\Lambda}^*_{Dk}\mathbf{\Lambda}^{*T}_{Dl} = \mathbf{\Lambda}^*_{Wk}\mathbf{\Lambda}^{*T}_{Wl} \tag{A34}$$

Regarding to the aggregated parameters, from Equation A3, A4, A8, and A16, it can be shown that

$$\begin{aligned}\mathbf{\Sigma}_{Akl} = \mathbf{\Sigma}_{Bkl} + n^{-1}\mathbf{\Sigma}_{Wkl} &= \mathbf{D}^{1/2}_{Bk}(\mathbf{\Lambda}^*_{Bk}\mathbf{\Lambda}^{*T}_{Bl})\mathbf{D}^{1/2}_{Bl} + n^{-1}\mathbf{D}^{1/2}_{Wk}(\mathbf{\Lambda}^*_{Wk}\mathbf{\Lambda}^{*T}_{Wl})\mathbf{D}^{1/2}_{Wl} \\ &= \rho^{1/2}\mathbf{D}^{1/2}_{k}(\mathbf{\Lambda}^*_{Bk}\mathbf{\Lambda}^{*T}_{Bl})\rho^{1/2}\mathbf{D}^{1/2}_{l} \\ &+ n^{-1}(1-\rho)^{1/2}\mathbf{D}^{1/2}_{k}(\mathbf{\Lambda}^*_{Bk}\mathbf{\Lambda}^{*T}_{Bl})(1-\rho)^{1/2}\mathbf{D}^{1/2}_{l} \\ &= \left(\frac{\rho(n-1)+1}{n}\right)\mathbf{D}^{1/2}_{k}(\mathbf{\Lambda}^*_{Bk}\mathbf{\Lambda}^{*T}_{Bl})\mathbf{D}^{1/2}_{l}\end{aligned} \tag{A35}$$

From Equations A16 and A31, it can also be shown that

$$\mathbf{\Sigma}_{Akl} = \mathbf{D}^{1/2}_{Ak}(\mathbf{\Lambda}^*_{Ak}\mathbf{\Lambda}^{*T}_{Al})\mathbf{D}^{1/2}_{Al} = \left(\frac{\rho(n-1)+1}{n}\right)\mathbf{D}^{1/2}_{k}(\mathbf{\Lambda}^*_{Ak}\mathbf{\Lambda}^{*T}_{Al})\mathbf{D}^{1/2}_{l} \tag{A36}$$

From Equations A35 and A36,

$$\mathbf{\Lambda}^*_{Ak}\mathbf{\Lambda}^{*T}_{Al} = \mathbf{\Lambda}^*_{Bk}\mathbf{\Lambda}^{*T}_{Bl} \tag{A37}$$

From Equations A18, A28, and A34, the factor loading from disaggregation method will be

$$\begin{aligned}
\boldsymbol{\lambda}_{D2} &= \left(\mathbf{D}_{D1}^{1/2}(\boldsymbol{\Lambda}_{D1}^{*}\boldsymbol{\Lambda}_{D3}^{*T})\mathbf{D}_{D3}^{1/2}\mathbf{D}_{D3}^{1/2}(\boldsymbol{\Lambda}_{D3}^{*}\boldsymbol{\Lambda}_{D1}^{*T})\mathbf{D}_{D1}^{1/2}\right)^{-1}\mathbf{D}_{D1}^{1/2}(\boldsymbol{\Lambda}_{D1}^{*}\boldsymbol{\Lambda}_{D3}^{*T})\mathbf{D}_{D3}^{1/2}\mathbf{D}_{D3}^{1/2}(\boldsymbol{\Lambda}_{D3}^{*}\boldsymbol{\Lambda}_{D2}^{*T})\mathbf{D}_{D2}^{1/2} \\
&= \left(\frac{1}{1-\rho}\right)^{-2}\left(\mathbf{D}_{W1}^{1/2}(\boldsymbol{\Lambda}_{W1}^{*}\boldsymbol{\Lambda}_{W3}^{*T})\mathbf{D}_{W3}^{1/2}\mathbf{D}_{W3}^{1/2}(\boldsymbol{\Lambda}_{W3}^{*}\boldsymbol{\Lambda}_{W1}^{*T})\mathbf{D}_{W1}^{1/2}\right)^{-1}\left(\frac{1}{1-\rho}\right)^{2}\mathbf{D}_{W1}^{1/2}(\boldsymbol{\Lambda}_{W1}^{*}\boldsymbol{\Lambda}_{W3}^{*T})\mathbf{D}_{W3}^{1/2}\mathbf{D}_{W3}^{1/2}(\boldsymbol{\Lambda}_{W3}^{*}\boldsymbol{\Lambda}_{W2}^{*T})\mathbf{D}_{W2}^{1/2} \\
&= \boldsymbol{\lambda}_{W2}
\end{aligned} \tag{A38}$$

Thus, the disaggregated factor loading is equal to the micro-level factor loading or $\boldsymbol{\Lambda}_D = \boldsymbol{\Lambda}_W$. By a similar method, the aggregated factor loading can be shown to equal the macro-level factor loading, $\boldsymbol{\Lambda}_A = \boldsymbol{\Lambda}_B$. From Equations A19-A22, the disaggregated unique variance will be

$$\mathbf{K}_D = \boldsymbol{\Lambda}_D(\boldsymbol{\Lambda}_D'\boldsymbol{\Lambda}_D)^{-\mathbf{1}}\boldsymbol{\Lambda}_D' = \boldsymbol{\Lambda}_W(\boldsymbol{\Lambda}_W'\boldsymbol{\Lambda}_W)^{-\mathbf{1}}\boldsymbol{\Lambda}_W' = \mathbf{K}_W \tag{A39}$$

$$\mathbf{E}_D = \mathbf{I} - (\mathbf{K}_D * \mathbf{K}_D) = \mathbf{I} - (\mathbf{K}_W * \mathbf{K}_W) = \mathbf{E}_W \tag{A40}$$

$$\begin{aligned}
\mathbf{g}_D = \mathrm{diag}(\boldsymbol{\Sigma}_D - \mathbf{K}_D\boldsymbol{\Sigma}_D\mathbf{K}_D) &= \mathrm{diag}\left(\left(\frac{1}{1-\rho}\right)\boldsymbol{\Sigma}_W - \mathbf{K}_W\left(\frac{1}{1-\rho}\right)\boldsymbol{\Sigma}_W\mathbf{K}_W\right) \\
&= \left(\frac{1}{1-\rho}\right)\mathrm{diag}(\boldsymbol{\Sigma}_W - \mathbf{K}_W\boldsymbol{\Sigma}_W\mathbf{K}_W) = \left(\frac{1}{1-\rho}\right)\mathbf{g}_W
\end{aligned} \tag{A41}$$

$$\mathrm{diag}(\boldsymbol{\Theta}_D) = \mathbf{E}_D^{-1}\mathbf{g}_D = \mathbf{E}_W^{-1}\left(\frac{1}{1-\rho}\right)\mathbf{g}_W = \left(\frac{1}{1-\rho}\right)\mathrm{diag}(\boldsymbol{\Theta}_W) \tag{A42}$$

Because the off-diagonal elements of $\boldsymbol{\Theta}_D$ and $\boldsymbol{\Theta}_W$ are 0, $\boldsymbol{\Theta}_D = \left(\frac{1}{1-\rho}\right)\boldsymbol{\Theta}_W$. By a similar method, it can be shown that $\boldsymbol{\Theta}_A = \left(\frac{\rho(n-1)+1}{\rho n}\right)\boldsymbol{\Theta}_B$. From Equation A23, the disaggregated covariance matrix between factors will be

$$\begin{aligned}
\boldsymbol{\Psi}_D &= (\boldsymbol{\Lambda}_D^T\boldsymbol{\Lambda}_D)^{-\mathbf{1}}\boldsymbol{\Lambda}_D^T(\boldsymbol{\Sigma}_D - \boldsymbol{\Theta}_D)\boldsymbol{\Lambda}_D(\boldsymbol{\Lambda}_D^T\boldsymbol{\Lambda}_D)^{-1} \\
&= (\boldsymbol{\Lambda}_W^T\boldsymbol{\Lambda}_W)^{-\mathbf{1}}\boldsymbol{\Lambda}_W^T\left(\left(\frac{1}{1-\rho}\right)\boldsymbol{\Sigma}_W - \left(\frac{1}{1-\rho}\right)\boldsymbol{\Theta}_W\right)\boldsymbol{\Lambda}_W(\boldsymbol{\Lambda}_W^T\boldsymbol{\Lambda}_W)^{-\mathbf{1}} \\
&= \left(\frac{1}{1-\rho}\right)\boldsymbol{\Psi}_W
\end{aligned} \tag{A43}$$

By a similar method, it can be shown that $\boldsymbol{\Psi}_A = \left(\frac{\rho(n-1)+1}{\rho n}\right)\boldsymbol{\Psi}_B$. From Equation A24 and A43, the disaggregated factors' variances will be

$$\mathbf{D}_{D\eta} = \text{diag}(\boldsymbol{\Psi}_D) = \text{diag}\left(\left(\frac{1}{1-\rho}\right)\boldsymbol{\Psi}_W\right) = \frac{1}{1-\rho}\mathbf{D}_{W\eta} \tag{A44}$$

By a similar method, it can be shown that $\mathbf{D}_{A\eta} = \frac{\rho(n-1)+1}{\rho n}\mathbf{D}_{B\eta}$. From Equations A28 and A29, it can be shown that $\mathbf{D}_{DY} = \frac{1}{1-\rho}\mathbf{D}_{WY}$ and $\mathbf{D}_{AY} = \frac{\rho(n-1)+1}{\rho n}\mathbf{D}_{BY}$. Therefore, from Equations A26 and A27, the disaggregated standardized factor loading and factor correlation will be

$$\boldsymbol{\Lambda}_D^* = \mathbf{D}_{DY}^{-1/2}\boldsymbol{\Lambda}_D\mathbf{D}_{D\eta}^{1/2} = \left(\frac{1}{1-\rho}\mathbf{D}_{WY}\right)^{-1/2}\boldsymbol{\Lambda}_W\left(\frac{1}{1-\rho}\mathbf{D}_{W\eta}\right)^{1/2} = \mathbf{D}_{WY}^{-1/2}\boldsymbol{\Lambda}_W\mathbf{D}_{W\eta}^{1/2} = \boldsymbol{\Lambda}_W^* \tag{A45}$$

$$\begin{aligned}\boldsymbol{\Psi}_D^* = \mathbf{D}_{D\eta}^{-1/2}\boldsymbol{\Psi}_D\mathbf{D}_{D\eta}^{-1/2} &= \left(\frac{1}{1-\rho}\mathbf{D}_{W\eta}\right)^{-1/2}\left(\frac{1}{1-\rho}\right)\boldsymbol{\Psi}_W\left(\frac{1}{1-\rho}\mathbf{D}_{W\eta}\right)^{-1/2} \\ &= \mathbf{D}_{W\eta}^{-1/2}\boldsymbol{\Psi}_W\mathbf{D}_{W\eta}^{-1/2} = \boldsymbol{\Psi}_W^*\end{aligned} \tag{A46}$$

Therefore, the standardized factor loading and factor correlation are not deviated from the disaggregation method in this case. It can also be showed that $\boldsymbol{\Lambda}_A^* = \boldsymbol{\Lambda}_B^*$ and $\boldsymbol{\Psi}_A^* = \boldsymbol{\Psi}_B^*$. That is, the standardized factor loading and factor correlation are not deviated from the aggregation method.

Case 2: Standardized Metric Invariance

ICCs are equal across all indicators: $\mathbf{P} = \rho\mathbf{I}$. Factor structures are the same in both levels. Standardized factor loadings are proportional between the macro and micro levels: $\boldsymbol{\Lambda}_B^* = c\boldsymbol{\Lambda}_W^*$**,** where *c* is the proportion. By this constraint, the unstandardized factor loading can fixed to be equal across levels. Factor correlations are different across levels: $\boldsymbol{\Psi}_B^* \neq \boldsymbol{\Psi}_W^*$.

To find the disaggregated and aggregated factor loadings in terms of the micro and macro factor loadings, respectively, by using Equation A18, two intermediate steps are required. First, because $\mathbf{P} = \rho\mathbf{I}$, Equations A30 and A31 are still held in this case. Second, from Equation A3, A4, A7, and A16, it can be shown that

$$\boldsymbol{\Sigma}_{Dkl} = \boldsymbol{\Sigma}_{Bkl} + \boldsymbol{\Sigma}_{Wkl} = \mathbf{D}_{Bk}^{1/2}(\boldsymbol{\Lambda}_{Bk}^{*}\boldsymbol{\Lambda}_{Bl}^{*T})\mathbf{D}_{Bl}^{1/2} + \mathbf{D}_{Wk}^{1/2}(\boldsymbol{\Lambda}_{Wk}^{*}\boldsymbol{\Lambda}_{Wl}^{*T})\mathbf{D}_{Wl}^{1/2} \tag{A47}$$

$$= \rho^{1/2}\mathbf{D}_{k}^{1/2}(c^{-1}\boldsymbol{\Lambda}_{Wk}^{*}c^{-1}\boldsymbol{\Lambda}_{Wl}^{*T})\rho^{1/2}\mathbf{D}_{l}^{1/2}$$

$$+ (1-\rho)^{1/2}\mathbf{D}_{k}^{1/2}(\boldsymbol{\Lambda}_{Wk}^{*}\boldsymbol{\Lambda}_{Wl}^{*T})(1-\rho)^{1/2}\mathbf{D}_{l}^{1/2}$$

$$= \left(\frac{\rho + c^2 - c^2\rho}{c^2}\right)\mathbf{D}_{k}^{1/2}(\boldsymbol{\Lambda}_{Wk}^{*}\boldsymbol{\Lambda}_{Wl}^{*T})\mathbf{D}_{l}^{1/2}$$

From Equations A16 and A30, it can also be shown that

$$\boldsymbol{\Sigma}_{Dkl} = \mathbf{D}_{Dk}^{1/2}(\boldsymbol{\Lambda}_{Dk}^{*}\boldsymbol{\Lambda}_{Dl}^{*T})\mathbf{D}_{Dl}^{1/2} = \mathbf{D}_{k}^{1/2}(\boldsymbol{\Lambda}_{Dk}^{*}\boldsymbol{\Lambda}_{Dl}^{*T})\mathbf{D}_{l}^{1/2} \tag{A48}$$

From Equations A47 and A48,

$$\boldsymbol{\Lambda}_{Dk}^{*}\boldsymbol{\Lambda}_{Dl}^{*T} = \left(\frac{\rho + c^2 - c^2\rho}{c^2}\right)\boldsymbol{\Lambda}_{Wk}^{*}\boldsymbol{\Lambda}_{Wl}^{*T} \tag{A49}$$

Regarding the aggregated parameters, from Equation A3, A4, A8, and A16, it can be shown that

$$\boldsymbol{\Sigma}_{Akl} = \boldsymbol{\Sigma}_{Bkl} + n^{-1}\boldsymbol{\Sigma}_{Wkl} = \mathbf{D}_{Bk}^{1/2}(\boldsymbol{\Lambda}_{Bk}^{*}\boldsymbol{\Lambda}_{Bl}^{*T})\mathbf{D}_{Bl}^{1/2} + n^{-1}\mathbf{D}_{Wk}^{1/2}(\boldsymbol{\Lambda}_{Wk}^{*}\boldsymbol{\Lambda}_{Wl}^{*T})\mathbf{D}_{Wl}^{1/2} \tag{A50}$$

$$= \rho^{1/2}\mathbf{D}_{k}^{1/2}(\boldsymbol{\Lambda}_{Bk}^{*}\boldsymbol{\Lambda}_{Bl}^{*T})\rho^{1/2}\mathbf{D}_{l}^{1/2}$$

$$+ n^{-1}(1-\rho)^{1/2}\mathbf{D}_{k}^{1/2}(c\boldsymbol{\Lambda}_{Bk}^{*}c\boldsymbol{\Lambda}_{Bl}^{*T})(1-\rho)^{1/2}\mathbf{D}_{l}^{1/2}$$

$$= \left(\frac{\rho(n-c^2)+c^2}{n}\right)\mathbf{D}_{k}^{1/2}(\boldsymbol{\Lambda}_{Bk}^{*}\boldsymbol{\Lambda}_{Bl}^{*T})\mathbf{D}_{l}^{1/2}$$

From Equations A16 and A31, it can also be shown that

$$\boldsymbol{\Sigma}_{Akl} = \mathbf{D}_{Ak}^{1/2}(\boldsymbol{\Lambda}_{Ak}^{*}\boldsymbol{\Lambda}_{Al}^{*T})\mathbf{D}_{Al}^{1/2} = \left(\frac{\rho(n-1)+1}{n}\right)\mathbf{D}_{k}^{1/2}(\boldsymbol{\Lambda}_{Ak}^{*}\boldsymbol{\Lambda}_{Al}^{*T})\mathbf{D}_{l}^{1/2} \tag{A51}$$

From Equations A50 and A51,

$$\boldsymbol{\Lambda}_{Ai}^{*}\boldsymbol{\Lambda}_{Aj}^{*T} = \left(\frac{\rho(n-c^2)+c^2}{\rho(n-1)+1}\right)\boldsymbol{\Lambda}_{Bi}^{*}\boldsymbol{\Lambda}_{Bj}^{*T} \tag{A52}$$

From Equations A18, A28, and A49, the factor loading from disaggregation method will be

$$\begin{aligned}\boldsymbol{\lambda}_{D2} &= \left(\mathbf{D}_{D1}^{1/2}(\boldsymbol{\Lambda}_{D1}^{*}\boldsymbol{\Lambda}_{D3}^{*T})\mathbf{D}_{D3}^{1/2}\mathbf{D}_{D3}^{1/2}(\boldsymbol{\Lambda}_{D3}^{*}\boldsymbol{\Lambda}_{D1}^{*T})\mathbf{D}_{D1}^{1/2}\right)^{-1}\mathbf{D}_{D1}^{1/2}(\boldsymbol{\Lambda}_{D1}^{*}\boldsymbol{\Lambda}_{D3}^{*T})\mathbf{D}_{D3}^{1/2}\mathbf{D}_{D3}^{1/2}(\boldsymbol{\Lambda}_{D3}^{*}\boldsymbol{\Lambda}_{D2}^{*T})\mathbf{D}_{D2}^{1/2} \\ &= \left(\frac{\rho + c^2 - c^2\rho}{\mathrm{c}^2(1-\rho)}\right)^{-2}\left(\mathbf{D}_{W1}^{1/2}(\boldsymbol{\Lambda}_{W1}^{*}\boldsymbol{\Lambda}_{W3}^{*T})\mathbf{D}_{W3}^{1/2}\mathbf{D}_{W3}^{1/2}(\boldsymbol{\Lambda}_{W3}^{*}\boldsymbol{\Lambda}_{W1}^{*T})\mathbf{D}_{W1}^{1/2}\right)^{-1}\left(\frac{\rho + c^2 - c^2\rho}{\mathrm{c}^2(1-\rho)}\right)^{2}\mathbf{D}_{W1}^{1/2}(\boldsymbol{\Lambda}_{W1}^{*}\boldsymbol{\Lambda}_{W3}^{*T})\mathbf{D}_{W3}^{1/2}\mathbf{D}_{W3}^{1/2}(\boldsymbol{\Lambda}_{W3}^{*}\boldsymbol{\Lambda}_{W2}^{*T})\mathbf{D}_{W2}^{1/2} \\ &= \boldsymbol{\lambda}_{W2}\end{aligned} \tag{A53}$$

Thus, the disaggregated factor loading is equal to the micro-level factor loading or $\boldsymbol{\Lambda}_D = \boldsymbol{\Lambda}_W$. By a similar method, the aggregated factor loading can be shown to equal the macro-level factor loading, $\boldsymbol{\Lambda}_A = \boldsymbol{\Lambda}_B$. Therefore, based on the same logic of Equations A39 and A40, $\mathbf{K}_D = \mathbf{K}_W$, $\mathbf{E}_D = \mathbf{E}_W$, $\mathbf{K}_A = \mathbf{K}_B$, and $\mathbf{E}_A = \mathbf{E}_B$. Then, from Equations A7, A8, and A21,

$$\begin{aligned}\mathbf{g}_D = \operatorname{diag}(\boldsymbol{\Sigma}_D - \mathbf{K}_D\boldsymbol{\Sigma}_D\mathbf{K}_D) &= \operatorname{diag}(\boldsymbol{\Sigma}_B - \mathbf{K}_D\boldsymbol{\Sigma}_B\mathbf{K}_D + \boldsymbol{\Sigma}_W - \mathbf{K}_D\boldsymbol{\Sigma}_W\mathbf{K}_D) \\ &= \operatorname{diag}(\boldsymbol{\Sigma}_B - \mathbf{K}_D\boldsymbol{\Sigma}_B\mathbf{K}_D) + \operatorname{diag}(\boldsymbol{\Sigma}_W - \mathbf{K}_D\boldsymbol{\Sigma}_W\mathbf{K}_D) = \mathbf{g}_B + \mathbf{g}_W\end{aligned} \tag{A54}$$

$$\begin{aligned}\mathbf{g}_A = \operatorname{diag}(\boldsymbol{\Sigma}_A - \mathbf{K}_A\boldsymbol{\Sigma}_A\mathbf{K}_A) &= \operatorname{diag}(\boldsymbol{\Sigma}_B - \mathbf{K}_D\boldsymbol{\Sigma}_B\mathbf{K}_D + n^{-1}\boldsymbol{\Sigma}_W - n^{-1}\mathbf{K}_D\boldsymbol{\Sigma}_W\mathbf{K}_D) \\ &= \operatorname{diag}(\boldsymbol{\Sigma}_B - \mathbf{K}_D\boldsymbol{\Sigma}_B\mathbf{K}_D) + n^{-1}\operatorname{diag}(\boldsymbol{\Sigma}_W - \mathbf{K}_D\boldsymbol{\Sigma}_W\mathbf{K}_D) = \mathbf{g}_B + n^{-1}\mathbf{g}_W\end{aligned} \tag{A55}$$

From Equations A22, A54 and A55,

$$\operatorname{diag}(\boldsymbol{\Theta}_D) = \mathbf{E}_D^{-1}\mathbf{g}_D = \mathbf{E}_W^{-1}(\mathbf{g}_B + \mathbf{g}_W) = \operatorname{diag}(\boldsymbol{\Theta}_B) + \operatorname{diag}(\boldsymbol{\Theta}_W) \tag{A56}$$

$$\operatorname{diag}(\boldsymbol{\Theta}_A) = \mathbf{E}_A^{-1}\mathbf{g}_A = \mathbf{E}_W^{-1}(\mathbf{g}_B + n^{-1}\mathbf{g}_W) = \operatorname{diag}(\boldsymbol{\Theta}_B) + n^{-1}\operatorname{diag}(\boldsymbol{\Theta}_W) \tag{A57}$$

Because the off-diagonal elements of $\boldsymbol{\Theta}_B$, $\boldsymbol{\Theta}_W$, $\boldsymbol{\Theta}_D$, and $\boldsymbol{\Theta}_A$ are 0, then $\boldsymbol{\Theta}_D = \boldsymbol{\Theta}_B + \boldsymbol{\Theta}_W$ and $\boldsymbol{\Theta}_A = \boldsymbol{\Theta}_B + n^{-1}\boldsymbol{\Theta}_W$. From Equation A23,

$$\begin{aligned}\boldsymbol{\Psi}_D &= (\boldsymbol{\Lambda}_D^T\boldsymbol{\Lambda}_D)^{-1}\boldsymbol{\Lambda}_D^T(\boldsymbol{\Sigma}_D - \boldsymbol{\Theta}_D)\boldsymbol{\Lambda}_D(\boldsymbol{\Lambda}_D'\boldsymbol{\Lambda}_D)^{-1} \\ &= (\boldsymbol{\Lambda}_W^T\boldsymbol{\Lambda}_W)^{-\mathbf{1}}\boldsymbol{\Lambda}_W^T\big(\boldsymbol{\Sigma}_B + \boldsymbol{\Sigma}_W - (\boldsymbol{\Theta}_B + \boldsymbol{\Theta}_W)\big)\boldsymbol{\Lambda}_W(\boldsymbol{\Lambda}_W^T\boldsymbol{\Lambda}_W)^{-\mathbf{1}} \\ &= \boldsymbol{\Psi}_B + \boldsymbol{\Psi}_W\end{aligned} \tag{A58}$$

$$\begin{aligned}\boldsymbol{\Psi}_A &= (\boldsymbol{\Lambda}_A^T\boldsymbol{\Lambda}_A)^{-1}\boldsymbol{\Lambda}_A^T(\boldsymbol{\Sigma}_A - \boldsymbol{\Theta}_A)\boldsymbol{\Lambda}_A(\boldsymbol{\Lambda}_A'\boldsymbol{\Lambda}_A)^{-1} \\ &= (\boldsymbol{\Lambda}_B^T\boldsymbol{\Lambda}_B)^{-\mathbf{1}}\boldsymbol{\Lambda}_B^T\big(\boldsymbol{\Sigma}_B + n^{-1}\boldsymbol{\Sigma}_W - (\boldsymbol{\Theta}_B + n^{-1}\boldsymbol{\Theta}_W)\big)\boldsymbol{\Lambda}_B(\boldsymbol{\Lambda}_B^T\boldsymbol{\Lambda}_B)^{-\mathbf{1}} \\ &= \boldsymbol{\Psi}_B + n^{-1}\boldsymbol{\Psi}_W\end{aligned} \tag{A59}$$

Because $\mathbf{D}_{D\eta} = diag(\boldsymbol{\Psi}_B + \boldsymbol{\Psi}_W)$, $\mathbf{D}_{A\eta} = diag(\boldsymbol{\Psi}_B + n^{-1}\boldsymbol{\Psi}_W)$, $\mathbf{D}_{DY} = diag(\boldsymbol{\Sigma}_B + \boldsymbol{\Sigma}_W)$, and $\mathbf{D}_{AY} = diag(\boldsymbol{\Sigma}_B + n^{-1}\boldsymbol{\Sigma}_W)$, from Equation A26,

$$\boldsymbol{\Lambda}_D^* = \mathbf{D}_{DY}^{-1/2}\boldsymbol{\Lambda}_D\mathbf{D}_{D\eta}^{1/2} = \mathbf{D}_{DY}^{-1/2}\boldsymbol{\Lambda}_W\mathbf{D}_{D\eta}^{1/2} = \mathbf{D}_{DY}^{-1/2}\mathbf{D}_{WY}^{1/2}\boldsymbol{\Lambda}_W^*\mathbf{D}_{W\eta}^{-1/2}\mathbf{D}_{D\eta}^{1/2} \quad \text{(A60)}$$

$$\boldsymbol{\Lambda}_A^* = \mathbf{D}_{AY}^{-1/2}\boldsymbol{\Lambda}_A\mathbf{D}_{A\eta}^{1/2} = \mathbf{D}_{AY}^{-1/2}\boldsymbol{\Lambda}_B\mathbf{D}_{A\eta}^{1/2} = \mathbf{D}_{AY}^{-1/2}\mathbf{D}_{BY}^{1/2}\boldsymbol{\Lambda}_B^*\mathbf{D}_{B\eta}^{-1/2}\mathbf{D}_{A\eta}^{1/2} \quad \text{(A61)}$$

Because all **D** matrices are diagonal, the disaggregated and aggregated standardized factor loadings for indicator *r* on factor *s* are

$$\lambda_{Drs}^* = \sqrt{\frac{\sigma_{Wrr}}{\sigma_{Wrr} + \sigma_{Brr}}}\lambda_{Wrs}^*\sqrt{\frac{\psi_{Wss} + \psi_{Bss}}{\psi_{Wss}}} \quad \text{(A62)}$$

$$\lambda_{Ars}^* = \sqrt{\frac{\sigma_{Brr}}{\frac{\sigma_{Wrr}}{n} + \sigma_{Brr}}}\lambda_{Brs}^*\sqrt{\frac{\frac{\psi_{Wss}}{n} + \psi_{Bss}}{\psi_{Bss}}} \quad \text{(A63)}$$

where σ_{Wrr} and σ_{Brr} are the micro- and macro-level indicator variances of variable *r*, ψ_{Wss} and ψ_{Bss} are the micro- and macro-level factor variance of factor *s*, and λ_{Wrs}^*, λ_{Brs}^*, λ_{Drs}^*, and λ_{Ars}^* are the micro-, macro-, disaggregated, and aggregated standardized factor loadings, respectively.

Equation A26 can be simplified as

$$\lambda_{rs}^* = \frac{\lambda_{rs}\psi_{ss}}{\sigma_{rr}} \quad \text{(A64)}$$

Therefore, Equations A62 and A63 can be transformed as

$$\lambda_{Drs}^{*} = \sqrt{\frac{\sigma_{Wrr}}{\sigma_{Wrr} + \sigma_{Brr}}}\lambda_{Wrs}^{*}\sqrt{\frac{\psi_{Wss} + \psi_{Bss}}{\psi_{Wss}}}$$

$$= \sqrt{\frac{\sigma_{Wrr}}{\sigma_{Wrr} + \sigma_{Brr}}}\lambda_{Wrs}^{*}\sqrt{\frac{\frac{\lambda_{Wrs}^{*}\sigma_{Wrr}}{\lambda_{Wrs}} + \frac{\lambda_{Brs}^{*}\sigma_{Brr}}{\lambda_{Brs}}}{\frac{\lambda_{Wrs}^{*}\sigma_{Wrr}}{\lambda_{Wrs}}}}$$

$$= \sqrt{\frac{\sigma_{Wrr}}{\sigma_{Wrr} + \sigma_{Brr}}}\lambda_{Wrs}^{*}\sqrt{\frac{\lambda_{Wrs}^{*}\sigma_{Wrr} + \lambda_{Brs}^{*}\sigma_{Brr}}{\lambda_{Wrs}^{*}\sigma_{Wrr}}}$$

$$= \sqrt{\frac{\lambda_{Wrs}^{*}\left(\lambda_{Wrs}^{*}\sigma_{Wrr} + \lambda_{Brs}^{*}\sigma_{Brr}\right)}{\sigma_{Wrr} + \sigma_{Brr}}} \quad \text{(A65)}$$

$$\lambda_{Ars}^{*} = \sqrt{\frac{\sigma_{Brr}}{\frac{\sigma_{Wrr}}{n} + \sigma_{Brr}}}\lambda_{Brs}^{*}\sqrt{\frac{\frac{\psi_{Wss}}{n} + \psi_{Bss}}{\psi_{Bss}}}$$

$$= \sqrt{\frac{\sigma_{Brr}}{\frac{\sigma_{Wrr}}{n} + \sigma_{Brr}}}\lambda_{Brs}^{*}\sqrt{\frac{\frac{\lambda_{Wrs}^{*}\sigma_{Wrr}}{n\lambda_{Wrs}} + \frac{\lambda_{Brs}^{*}\sigma_{Brr}}{\lambda_{Brs}}}{\frac{\lambda_{Brs}^{*}\sigma_{Brr}}{\lambda_{Brs}}}}$$

$$= \sqrt{\frac{\sigma_{Brr}}{\frac{\sigma_{Wrr}}{n} + \sigma_{Brr}}}\lambda_{Brs}^{*}\sqrt{\frac{\frac{\lambda_{Wrs}^{*}\sigma_{Wrr}}{n} + \lambda_{Brs}^{*}\sigma_{Brr}}{\lambda_{Brs}^{*}\sigma_{Brr}}}$$

$$= \sqrt{\frac{\lambda_{Brs}^{*}\left(\lambda_{Wrs}^{*}\frac{\sigma_{Wrr}}{n} + \lambda_{Brs}^{*}\sigma_{Brr}\right)}{\frac{\sigma_{Wrr}}{n} + \sigma_{Brr}}} \quad \text{(A66)}$$

From Equation A64, if $\lambda_{Wrs}^{*} = \lambda_{Brs}^{*}$, then $\lambda_{Drs}^{*} = \lambda_{Wrs}^{*}$. If $\lambda_{Wrs}^{*} < \lambda_{Brs}^{*}$, then $\lambda_{Drs}^{*} > \lambda_{Wrs}^{*}$. If $\lambda_{Wrs}^{*} > \lambda_{Brs}^{*}$, then $\lambda_{Drs}^{*} < \lambda_{Wrs}^{*}$. Thus, the difference of the disaggregated standardized loading is in the direction of the macro-level standardized loading. If σ_{Brr} or ICC is high, the degree of differences in the disaggregated standardized loading is higher. From Equation A65, if $\lambda_{Wrs}^{*} = \lambda_{Brs}^{*}$, then $\lambda_{Ars}^{*} = \lambda_{Brs}^{*}$. If $\lambda_{Wrs}^{*} < \lambda_{Brs}^{*}$, then $\lambda_{Ars}^{*} < \lambda_{Brs}^{*}$. If $\lambda_{Wrs}^{*} > \lambda_{Brs}^{*}$, then $\lambda_{Ars}^{*} > \lambda_{Brs}^{*}$. The difference in the aggregated standardized loading is in the direction of the micro-level standardized loading. If $\frac{\sigma_{Wrr}}{n}$ is high, then the degree of differences in the aggregated standardized loading is higher. $\frac{\sigma_{Wrr}}{n}$ is higher when ICC is low or the micro-level sample size is low. Therefore, the degree of differences in the aggregated standardized loading is higher when ICC is low or the micro-level sample size is low.

From Equation A27,

$$\begin{aligned}\mathbf{\Psi}_D^* &= \mathbf{D}_{D\eta}^{-1/2}\mathbf{\Psi}_D\mathbf{D}_{D\eta}^{-1/2} = \mathbf{D}_{D\eta}^{-1/2}(\mathbf{\Psi}_B + \mathbf{\Psi}_W)\mathbf{D}_{D\eta}^{-1/2} \\ &= \mathbf{D}_{D\eta}^{-1/2}\mathbf{\Psi}_B\mathbf{D}_{D\eta}^{-1/2} + \mathbf{D}_{D\eta}^{-1/2}\mathbf{\Psi}_W\mathbf{D}_{D\eta}^{-1/2} \\ &= \mathbf{D}_{D\eta}^{-1/2}\mathbf{D}_{B\eta}^{1/2}\mathbf{\Psi}_B^*\mathbf{D}_{B\eta}^{1/2}\mathbf{D}_{D\eta}^{-1/2} + \mathbf{D}_{D\eta}^{-1/2}\mathbf{D}_{W\eta}^{1/2}\mathbf{\Psi}_W^*\mathbf{D}_{W\eta}^{1/2}\mathbf{D}_{D\eta}^{-1/2} \end{aligned} \quad \text{(A67)}$$

$$\begin{aligned}\mathbf{\Psi}_A^* &= \mathbf{D}_{A\eta}^{-1/2}\mathbf{\Psi}_A\mathbf{D}_{A\eta}^{-1/2} = \mathbf{D}_{A\eta}^{-1/2}\mathbf{\Psi}_B\mathbf{D}_{A\eta}^{-1/2} + n^{-1}\mathbf{D}_{A\eta}^{-1/2}\mathbf{\Psi}_W\mathbf{D}_{A\eta}^{-1/2} \\ &= \mathbf{D}_{A\eta}^{-1/2}\mathbf{D}_{B\eta}^{1/2}\mathbf{\Psi}_B^*\mathbf{D}_{B\eta}^{1/2}\mathbf{D}_{A\eta}^{-1/2} + n^{-1}\mathbf{D}_{A\eta}^{-1/2}\mathbf{D}_{W\eta}^{1/2}\mathbf{\Psi}_W^*\mathbf{D}_{W\eta}^{1/2}\mathbf{D}_{A\eta}^{-1/2}\end{aligned} \quad \text{(A68)}$$

Because all **D** matrices are diagonal, the disaggregated and aggregated factor correlations between factors *s* and *t* when $s \neq t$ are

$$\psi_{Dst}^* = \sqrt{\frac{\psi_{Bss}}{\psi_{Bss} + \psi_{Wss}}}\psi_{Bst}^*\sqrt{\frac{\psi_{Btt}}{\psi_{Btt} + \psi_{Wtt}}} + \sqrt{\frac{\psi_{Wss}}{\psi_{Bss} + \psi_{Wss}}}\psi_{Wst}^*\sqrt{\frac{\psi_{Wtt}}{\psi_{Btt} + \psi_{Wtt}}} \quad \text{(A69)}$$

$$\psi_{Ast}^* = \sqrt{\frac{\psi_{Bss}}{\psi_{Bss} + \frac{\psi_{Wss}}{n}}}\psi_{Bst}^*\sqrt{\frac{\psi_{Btt}}{\psi_{Btt} + \frac{\psi_{Wtt}}{n}}} + \sqrt{\frac{\frac{\psi_{Wss}}{n}}{\psi_{Bss} + \frac{\psi_{Wss}}{n}}}\frac{\psi_{Wst}^*}{n}\sqrt{\frac{\frac{\psi_{Wtt}}{n}}{\psi_{Btt} + \frac{\psi_{Wtt}}{n}}} \quad \text{(A70)}$$

where ψ_{Wst}^*, ψ_{Bst}^*, ψ_{Dst}^*, and ψ_{Ast}^* are the micro-, macro-, disaggregated, and aggregated factor correlations, respectively. From Equations A69 and A70, obviously, the disaggregated and aggregated factor correlations are the weighted average of the both level factor correlations. Therefore, the difference of the disaggregated factor correlations is in the direction of the macro-level correlations and the difference of the aggregated factor correlations is in the direction of the micro-level correlations.

Case 3: True Metric Invariance

ICCs are equal across all indicators, $\mathbf{P} = \rho\mathbf{I}$. Factor structures are the same in both levels. Unstandardized factor loadings are equal across levels: $\mathbf{\Lambda}_B = \mathbf{\Lambda}_W$. Factor covariances are different across levels: $\mathbf{\Psi}_B \neq \mathbf{\Psi}_W$.

Let *k* and *l* be 1, 2, or 3 and $k \neq l$. We will use Equation A17 to equate the disaggregated and aggregated factor loadings to the micro and macro level factor loadings, respectively. Thus, we need to make $\mathbf{\Sigma}_{Dkl}$ and $\mathbf{\Sigma}_{Akl}$ in terms of $\mathbf{\Sigma}_{Wkl}$ and $\mathbf{\Sigma}_{Bkl}$, respectively. The reduced covariance matrix can be written in terms of unstandardized loadings and factor variance (ψ):

$$\begin{bmatrix} \mathbf{\Sigma_{11}} & \mathbf{\Sigma_{12}} & \mathbf{\Sigma_{13}} \\ \mathbf{\Sigma_{21}} & \mathbf{\Sigma_{22}} & \mathbf{\Sigma_{23}} \\ \mathbf{\Sigma_{31}} & \mathbf{\Sigma_{32}} & \mathbf{\Sigma_{33}} \end{bmatrix} = \begin{bmatrix} \psi\mathbf{\Lambda}_1\mathbf{\Lambda}_1^T + \mathbf{\Theta}_1 & \psi\mathbf{\Lambda}_1\mathbf{\Lambda}_2^T & \psi\mathbf{\Lambda}_1\mathbf{\Lambda}_3^T \\ \psi\mathbf{\Lambda}_2\mathbf{\Lambda}_1^T & \psi\mathbf{\Lambda}_2\mathbf{\Lambda}_2^T + \mathbf{\Theta}_2 & \psi\mathbf{\Lambda}_2\mathbf{\Lambda}_3^T \\ \psi\mathbf{\Lambda}_3\mathbf{\Lambda}_1^T & \psi\mathbf{\Lambda}_3\mathbf{\Lambda}_2^T & \psi\mathbf{\Lambda}_3\mathbf{\Lambda}_3^T + \mathbf{\Theta}_3 \end{bmatrix} \tag{A71}$$

Therefore,

$$\mathbf{\Sigma}_{kl} = \psi\mathbf{\Lambda}_k\mathbf{\Lambda}_l^T \tag{A72}$$

Then, $\mathbf{\Sigma}_{Dkl}$ and $\mathbf{\Sigma}_{Akl}$ can be written in terms of $\mathbf{\Sigma}_{Wkl}$ and $\mathbf{\Sigma}_{Bkl}$:

$$\begin{aligned} \mathbf{\Sigma}_{Dkl} &= \mathbf{\Sigma}_{Bkl} + \mathbf{\Sigma}_{Wkl} = \psi_B\mathbf{\Lambda}_{Bk}\mathbf{\Lambda}_{Bl}^T + \psi_W\mathbf{\Lambda}_{Wk}\mathbf{\Lambda}_{Wl}^T = (\psi_B + \psi_W)\mathbf{\Lambda}_{Wk}\mathbf{\Lambda}_{Wl}^T \\ &= \frac{\psi_B + \psi_W}{\psi_W}\,\mathbf{\Sigma}_{Wkl} \end{aligned} \tag{A73}$$

$$\begin{aligned} \mathbf{\Sigma}_{Akl} &= \mathbf{\Sigma}_{Bkl} + n^{-1}\mathbf{\Sigma}_{Wkl} = \psi_B\mathbf{\Lambda}_{Bk}\mathbf{\Lambda}_{Bl}^T + \frac{\psi_W}{n}\mathbf{\Lambda}_{Wk}\mathbf{\Lambda}_{Wl}^T = \left(\psi_B + \frac{\psi_W}{n}\right)\mathbf{\Lambda}_{Bi}\mathbf{\Lambda}_{Bj}^T \\ &= \frac{\psi_B + \frac{\psi_W}{n}}{\psi_B}\,\mathbf{\Sigma}_{Wkl} \end{aligned} \tag{A74}$$

From Equation A17, the disaggregated factor loading will be

$$\boldsymbol{\lambda}_{D2} = (\boldsymbol{\Sigma}_{D13}\boldsymbol{\Sigma}_{D31})^{-1}\boldsymbol{\Sigma}_{D13}\boldsymbol{\Sigma}_{D32}$$
$$= \left(\frac{\psi_B + \psi_W}{\psi_W}\boldsymbol{\Sigma}_{W13}\frac{\psi_B + \psi_W}{\psi_W}\boldsymbol{\Sigma}_{W31}\right)^{-1}\frac{\psi_B + \psi_W}{\psi_W}\boldsymbol{\Sigma}_{W13}\frac{\psi_B + \psi_W}{\psi_W}\boldsymbol{\Sigma}_{W32} = \boldsymbol{\lambda}_{W2} \quad \text{(A75)}$$

Thus, the disaggregated factor loading is equal to the micro-level factor loading or $\boldsymbol{\Lambda}_D = \boldsymbol{\Lambda}_W$. By a similar method, the aggregated factor loading is equal to the macro-level factor loading or $\boldsymbol{\Lambda}_A = \boldsymbol{\Lambda}_B$. Because $\mathbf{P} = \rho\mathbf{I}$, Equations A30 and A31 still hold in this case as well. Follow the same logic in Equations A54-A70. The Equations A65, A66, A69, and A70 are true in this case.

Impact of Ignoring Nesting on Maximal Reliability

Maximal reliability (*H*) is the reliability of the optimal linear combinations of observed scores across items (Raykov, 2004). We will consider maximal reliability first because it is a direct function of standardized loadings. Then, we will show the impact of ignoring clustering on coefficients alpha or omega that are functions of unstandardized coefficients. Maximal reliability of a scale *s* (H_s) is defined:

$$H_s = \frac{\sum_{r=1}^{p}\frac{(\lambda_{rs}^*)^2}{1-(\lambda_{rs}^*)^2}}{1+\sum_{r=1}^{p}\frac{(\lambda_{rs}^*)^2}{1-(\lambda_{rs}^*)^2}}. \quad \text{(A76)}$$

Assuming that standardized loadings are equal across indicators, it can be shown that

$$\frac{H_s}{1-H_s} = p\frac{(\lambda_{rs}^*)^2}{1-(\lambda_{rs}^*)^2}. \quad \text{(A77)}$$

Under Cases 2 and 3, assuming that micro-level observed variances, and macro-level observed variances are equal across indicators, Equation A65 can be transformed as

$$(\lambda_{Drs}^*)^2 = (\lambda_{Wrs}^*)^2\left(\frac{\sigma_{Wrr}}{\sigma_{Wrr}+\sigma_{Brr}}\right) + \lambda_{Wrs}^*\lambda_{Brs}^*\left(\frac{\sigma_{Brr}}{\sigma_{Wrr}+\sigma_{Brr}}\right). \quad \text{(A78)}$$

Because $\sigma_{Wrr}, \sigma_{Brr} \geq 0$, then the lower and upper bounds of $(\lambda_{Drs}^*)^2$ are λ_{Wrs}^2 and $\lambda_{Wrs}\lambda_{Brs}$.

In the following proof, we assume that $\lambda_{Brs} > \lambda_{Wrs}$ ($H_{Bs} > H_{Ws}$) first and we will show that the same logic can be applied for $\lambda_{Wrs} > \lambda_{Brs}$ ($H_{Ws} > H_{Bs}$). Then, from Equations A77 and A78, it can be shown that

$$(\lambda_{Wrs}^{*})^2 \leq (\lambda_{Drs}^{*})^2 \leq \lambda_{Wrs}^{*}\lambda_{Brs}^{*} \leq (\lambda_{Brs}^{*})^2. \quad \text{(A79)}$$

Because $0 \leq (\lambda_{Wrs}^{*})^2 \leq 1$, then

$$p\frac{(\lambda_{Wrs}^{*})^2}{1-(\lambda_{Wrs}^{*})^2} \leq p\frac{(\lambda_{Drs}^{*})^2}{1-(\lambda_{Drs}^{*})^2} \leq p\frac{(\lambda_{Brs}^{*})^2}{1-(\lambda_{Brs}^{*})^2}, \quad \text{(A80)}$$

or

$$\frac{H_{Ws}}{1-H_{Ws}} \leq \frac{H_{Ds}}{1-H_{Ds}} \leq \frac{H_{Bs}}{1-H_{Bs}}. \quad \text{(A81)}$$

Because $0 \leq H_s \leq 1$,

$$H_{Ws} \leq H_{Ds} \leq H_{Bs}. \quad \text{(A82)}$$

It can also be shown that, if $\lambda_{Brs} < \lambda_{Wrs}$ ($H_{Bs} < H_{Ws}$), then $H_{Bs} \leq H_{Ds} \leq H_{Ws}$. Therefore, disaggregated maximal reliability is in between the micro- and macro-level maximal reliabilities. Disaggregated maximal reliability will be greater or lower than micro-level maximal reliability if macro-level maximal reliability is higher or lower, respectively.

The logic of proofs on disaggregated maximal reliability can be applied to aggregated maximal reliability. That is, if $H_{Bs} < H_{Ws}$, then $H_{Bs} < H_{As} < H_{Ws}$, and vice versa. Therefore, the result will show that aggregated maximal reliability is in between the micro- and macro-level maximal reliabilities. Aggregated maximal reliability will be greater or lower than macro-level maximal reliability if micro-level maximal reliability is higher or lower, respectively.

Next, coefficient omega (ω_s) is the reliability of the unweighted composites of observed scores across items (Raykov, 1997; Raykov & Shrout, 2002):

$$\omega_s = \frac{\mathrm{Var}\left(\sum_{r=1}^{p} \lambda_{rs}\eta_s\right)}{\mathrm{Var}\left(\sum_{r=1}^{p} \lambda_{rs}\eta_s + \sum_{r=1}^{p} \varepsilon_r\right)} = \frac{\left(\sum_{r=1}^{p} \lambda_{rs}\right)^2 \psi_{ss}}{\left(\sum_{r=1}^{p} \lambda_{rs}\right)^2 \psi_{ss} + \sum_{r=1}^{p} \theta_{rr}}, \quad \text{(A83)}$$

where η_s is factor *s*, ε_r is measurement error of indictor *r*, λ_{rs} is unstandardized factor loading linking indicator *r* to factor *s*, ψ_{ss} is variance of factor *s*, and θ_{rr} is measurement error variance of indicator *r*. Assuming that observed variances are equal across indicators, from Equation A83, it can be shown that

$$\omega_s = \frac{\left(\sum_{r=1}^{p}\left(\lambda_{rs} \cdot \sqrt{\frac{\psi_{ss}}{\sigma_{rr}}}\right)\right)^2}{\left(\sum_{r=1}^{p}\left(\lambda_{rs} \cdot \sqrt{\frac{\psi_{ss}}{\sigma_{rr}}}\right)\right)^2 + \sum_{r=1}^{p}\frac{\theta_{rr}}{\sigma_{rr}}}, \quad \text{(A84)}$$

where σ_{rr} is observed variance of indicator *r*. Because $\lambda_{rs}^* = \lambda_{rs} \cdot \sqrt{\frac{\psi_{ss}}{\sigma_{rr}}}$ and $\frac{\theta_{rr}}{\sigma_{rr}} = 1 - (\lambda_{rs}^*)^2$, which represents uniqueness, coefficient omega of factor *s* will be

$$\omega_s = \frac{\left(\sum_{r=1}^{p} \lambda_{rs}^*\right)^2}{\left(\sum_{r=1}^{p} \lambda_{rs}^*\right)^2 + \sum_{r=1}^{p}(1 - (\lambda_{rs}^*)^2)}. \quad \text{(A85)}$$

Assuming that standardized loadings are equal across indicators, it can be shown that

$$\omega_s = \frac{p(\lambda_{rs}^*)^2}{p(\lambda_{rs}^*)^2 + 1 - (\lambda_{rs}^*)^2}, \quad \text{(A86)}$$

or

$$\frac{\omega_s}{1 - \omega_s} = p\frac{(\lambda_{rs}^*)^2}{1 - (\lambda_{rs}^*)^2}. \quad \text{(A87)}$$

Therefore, from Equation A77 and A87, it can be implied that $\omega_s = H_s$. In addition, because we assume standardized loadings and observed variances are equal across indicators, unstandardized loadings are also equal across indicators. In this case, coefficient omega is equal to coefficient alpha, as well as maximal reliability. Therefore, under Cases 2 and 3, assuming that micro-level observed variances, and macro-level observed variances are equal across indicators, the consequences of disaggregation and aggregation on coefficients and omega are the same as one of maximal reliability.

Online Appendix B

Mplus Example Code for Data Generation and Data Analysis

Data Generation and Analysis by the Full MCFA Model

```
TITLE: Group membership = 400/20; ICC = 0.55, 0.7, 0.85; h2 =0.49, 0.49, 0.49;
cor.between = 0.5; Full Multilevel
MONTECARLO: NAMES = Y1-Y6;
NOBSERVATIONS = 8000;
NREPS = 1000;
SEED = 12345;
NCSIZES = 1;
CSIZES = 400(20); ! Cluster size and the number of clusters
RESULTS = RESULTML.txt;
MODEL POPULATION:
%WITHIN%
YW1 BY Y1*0.670820393249937; ! Equation 8. sqrt(1 - ICC) * sd
YW2 BY Y2*0.547722557505166;
YW3 BY Y3*0.387298334620742;
YW4 BY Y4*0.670820393249937;
YW5 BY Y5*0.547722557505166;
YW6 BY Y6*0.387298334620742;
FW1 BY YW1*0.7 YW2*0.7 YW3*0.7; ! Micro-level standardized factor loading
FW2 BY YW4*0.7 YW5*0.7 YW6*0.7;
FW1-FW2*1;
FW1 WITH FW2*0.5; ! Micro-level factor correlation
YW1-YW6*0.51; ! Micro-level scaled unique variances
Y1-Y6@0;
%BETWEEN%
YB1 BY Y1*0.741619848709566; ! Equation 9, sqrt(ICC) * sd
YB2 BY Y2*0.836660026534076;
YB3 BY Y3*0.921954445729289;
YB4 BY Y4*0.741619848709566;
YB5 BY Y5*0.836660026534076;
YB6 BY Y6*0.921954445729289;
FB1 BY YB1*0.7 YB2*0.7 YB3*0.7; ! Macro-level standardized factor loading
FB2 BY YB4*0.7 YB5*0.7 YB6*0.7;
FB1-FB2*1;
FB1 WITH FB2*0.5; ! Macro-level factor correlation
YB1-YB6*0.51; ! Macro-level scaled unique variances
Y1-Y6@0;
MODEL:
%WITHIN%
YW1 BY Y1*0.670820393249937 (cw1);
YW2 BY Y2*0.547722557505166 (cw2);
YW3 BY Y3*0.387298334620742 (cw3);
YW4 BY Y4*0.670820393249937 (cw4);
YW5 BY Y5*0.547722557505166 (cw5);
YW6 BY Y6*0.387298334620742 (cw6);
FW1 BY YW1*0.7 (w1);
FW1 BY YW2*0.7 (w2);
FW1 BY YW3*0.7 (w3);
FW2 BY YW4*0.7 (w4);
FW2 BY YW5*0.7 (w5);
FW2 BY YW6*0.7 (w6);
FW1-FW2@1;
FW1 WITH FW2*0.5;
YW1*0.51 (ew1);
YW2*0.51 (ew2);
YW3*0.51 (ew3);
YW4*0.51 (ew4);
YW5*0.51 (ew5);
YW6*0.51 (ew6);
Y1-Y6@0;
%BETWEEN%
YB1 BY Y1*0.741619848709566 (cb1);
YB2 BY Y2*0.836660026534076 (cb2);
```

```
YB3 BY Y3*0.921954445729289 (cb3);
YB4 BY Y4*0.741619848709566 (cb4);
YB5 BY Y5*0.836660026534076 (cb5);
YB6 BY Y6*0.921954445729289 (cb6);
FB1 BY YB1*0.7 (b1);
FB1 BY YB2*0.7 (b2);
FB1 BY YB3*0.7 (b3);
FB2 BY YB4*0.7 (b4);
FB2 BY YB5*0.7 (b5);
FB2 BY YB6*0.7 (b6);
FB1-FB2@1;
FB1 WITH FB2*0.5;
YB1*0.51 (eb1);
YB2*0.51 (eb2);
YB3*0.51 (eb3);
YB4*0.51 (eb4);
YB5*0.51 (eb5);
YB6*0.51 (eb6);
Y1-Y6@0;
MODEL CONSTRAINT:
ew1 = 1 - w1^2; ! Model constraints in Equation 12
ew2 = 1 - w2^2;
ew3 = 1 - w3^2;
ew4 = 1 - w4^2;
ew5 = 1 - w5^2;
ew6 = 1 - w6^2;
eb1 = 1 - b1^2; ! Model constraints in Equation 13
eb2 = 1 - b2^2;
eb3 = 1 - b3^2;
eb4 = 1 - b4^2;
eb5 = 1 - b5^2;
eb6 = 1 - b6^2;
ANALYSIS:   TYPE = TWOLEVEL; ESTIMATOR = ML; ! Full information maximum likelihood
```

Data Analysis by the Partially Saturated Model

```
MODEL:
%WITHIN% ! The interpretation of these codes are the same as full MCFA model
YW1 BY Y1*0.670820393249937 (cw1);
YW2 BY Y2*0.547722557505166 (cw2);
YW3 BY Y3*0.387298334620742 (cw3);
YW4 BY Y4*0.670820393249937 (cw4);
YW5 BY Y5*0.547722557505166 (cw5);
YW6 BY Y6*0.387298334620742 (cw6);
FW1 BY YW1*0.7 (w1);
FW1 BY YW2*0.7 (w2);
FW1 BY YW3*0.7 (w3);
FW2 BY YW4*0.7 (w4);
FW2 BY YW5*0.7 (w5);
FW2 BY YW6*0.7 (w6);
FW1-FW2@1;
FW1 WITH FW2*0.5;
YW1*0.51 (ew1);
YW2*0.51 (ew2);
YW3*0.51 (ew3);
YW4*0.51 (ew4);
YW5*0.51 (ew5);
YW6*0.51 (ew6);
Y1-Y6@0;
%BETWEEN% ! Variances and covariances are saturated at the macro level
Y1 WITH Y2*0.304037004326776;
Y1 WITH Y3*0.335032461113845;
Y1 WITH Y4*0.13475;
Y1 WITH Y5*0.152018502163388;
Y1 WITH Y6*0.167516230556922;
Y2 WITH Y3*0.377967591203267;
Y2 WITH Y4*0.152018502163388;
Y2 WITH Y5*0.1715;
Y2 WITH Y6*0.188983795601634;
```

```
Y3 WITH Y4*0.167516230556922;
Y3 WITH Y5*0.188983795601634;
Y3 WITH Y6*0.20825;
Y4 WITH Y5*0.304037004326776;
Y4 WITH Y6*0.335032461113845;
Y5 WITH Y6*0.377967591203267;
Y1*0.55;
Y2*0.7;
Y3*0.85;
Y4*0.55;
Y5*0.7;
Y6*0.85;
MODEL CONSTRAINT:
ew1 = 1 - w1^2;
ew2 = 1 - w2^2;
ew3 = 1 - w3^2;
ew4 = 1 - w4^2;
ew5 = 1 - w5^2;
ew6 = 1 - w6^2;
ANALYSIS:   TYPE = TWOLEVEL; ESTIMATOR = ML;
```

Data Analysis by the Single-Level Model

```
MODEL:
%WITHIN%
YW1 BY Y1*0.670820393249937 (cw1); ! Equation 2. Standard deviation
YW2 BY Y2*0.547722557505166 (cw2);
YW3 BY Y3*0.387298334620742 (cw3);
YW4 BY Y4*0.670820393249937 (cw4);
YW5 BY Y5*0.547722557505166 (cw5);
YW6 BY Y6*0.387298334620742 (cw6);
FW1 BY YW1*0.7 (w1); ! Single-level standardized factor loading
FW1 BY YW2*0.7 (w2);
FW1 BY YW3*0.7 (w3);
FW2 BY YW4*0.7 (w4);
FW2 BY YW5*0.7 (w5);
FW2 BY YW6*0.7 (w6);
FW1-FW2@1;
FW1 WITH FW2*0.5; ! Single-level factor correlation
YW1*0.51 (ew1); ! Single-level scaled unique variances
YW2*0.51 (ew2);
YW3*0.51 (ew3);
YW4*0.51 (ew4);
YW5*0.51 (ew5);
YW6*0.51 (ew6);
Y1-Y6@0;
%BETWEEN% ! The between levels are not estimated.
Y1-Y6@0;
MODEL CONSTRAINT:
ew1 = 1 - w1^2; ! Model constraints in Equation 4
ew2 = 1 - w2^2;
ew3 = 1 - w3^2;
ew4 = 1 - w4^2;
ew5 = 1 - w5^2;
ew6 = 1 - w6^2;
ANALYSIS:   TYPE = TWOLEVEL; ESTIMATOR = ML;
```

Online Appendix C

Supplemental Results for Simulation 1

This section presents the supplemental results of the consequences of ignoring macro level similar in the first simulation study. Table C1 shows the eta-squared values of all main and interaction effects for all simulation conditions. Figure C1 shows results for convergence rate, rejection rate, and RMSEA. Figure C2 shows results for standardized loadings and their SEs. Figure C3 shows results for factor correlations and their SEs. Figures C1-C3 show results in the cases of equal ICC and equal communalities. Figures C4-C6 show results similar to Figures C1-C3 for the conditions with unequal ICCs. Figures C7-C9 show results for the conditions with unequal communalities. Figures C10-C12 show results for the conditions with unequal ICCs and unequal communalities.

Online Appendix D

Supplemental Results for Simulation 2

This section presents the supplemental results of the second simulation study showing the consequences of ignoring micro level when ICC or communalities are not equal across items within the same factor. Table D1 shows the eta-squared values of all main and interaction effects for all simulation conditions. Figure D1 shows results for convergence rate, rejection rate, and RMSEA. Figure D2 shows results for standardized loadings and their SEs. Figure D3 shows results for factor correlations and their SEs. Figures D1-D3 show results in the cases of equal ICC and equal communalities. Figures D4-D6 show results similar to Figures D1-D3 for the conditions with unequal ICCs. Figures D7-D9 show results for the conditions with unequal communalities. Figures D10-D12 show results for the conditions with unequal ICCs and unequal communalities.

Table C1

The full results of Eta-squared values for analysis of variance table of the simulation conditions for rejection rate (%sig), root mean square error of approximation (RMSEA), estimated micro-level standardized loadings, relative differences in standard error of micro-level standardized loadings, estimated micro-level factor correlation (CorW), and relative differences in micro-level factor correlation.

Effects	RR	RMSEA	Loading	RD SE Loading	CorW	RD SE CorW
N	.005	.005	.002	.011	.004	.029
ICC	.005	**.122**	.004	.006	.006	.009
ICCEQ	.000	.001	.003	.002	.000	.002
h^2	.000	.002	**.302**	**.270**	.000	**.083**
h^2EQ	.000	.000	.003	.000	.000	.003
CorB	.000	.000	.000	.002	**.295**	.025
Method	**.945**	**.619**	.010	.001	.004	.005
N : ICC	.006	.000	.001	.025	.013	**.078**
N : ICCEQ	.000	.000	.000	.000	.000	.004
ICC : ICCEQ	.000	.000	.002	.002	.000	.003
N : h^2	.000	.000	.001	.002	.002	.007
ICC : h^2	.000	.000	**.171**	**.131**	.004	**.067**
ICCEQ : h^2	.000	.000	.000	.000	.000	.000
N : h^2EQ	.000	.000	.000	.000	.000	.007
ICC : h^2EQ	.000	.000	.002	.001	.000	.007
ICCEQ : h^2EQ	.000	.000	.000	.000	.000	.000
h^2 : h^2EQ	.000	.000	.002	.001	.000	.000
N : CorB	.000	.000	.000	.002	.002	.003
ICC : CorB	.000	.000	.000	.004	**.140**	.016
ICCEQ : CorB	.000	.000	.000	.000	.000	.000
h^2 : CorB	.000	.000	.000	.001	.020	.001
h^2EQ : CorB	.000	.000	.000	.001	.000	.001
N : Method	.008	.014	.000	.019	.003	.013
ICC : Method	.003	**.226**	.006	.009	.003	.002
ICCEQ : Method	.000	.001	.003	.003	.000	.000
h^2 : Method	.000	.005	**.301**	**.237**	.000	**.061**
h^2EQ : Method	.000	.000	.001	.001	.000	.000
CorB : Method	.000	.000	.000	.005	**.264**	.020
N : ICC : ICCEQ	.000	.000	.000	.003	.000	.008
N : ICC : h^2	.001	.000	.001	.006	.010	**.098**
N : ICCEQ : h^2	.000	.000	.000	.000	.000	.001
ICC : ICCEQ : h^2	.000	.000	.000	.002	.000	.007
N : ICC : h^2EQ	.000	.000	.001	.007	.001	.019

N : ICCEQ : h^2EQ	.000	.000	.000	.000	.000	.001
ICC : ICCEQ : h^2EQ	.000	.000	.000	.000	.000	.001
N : h^2 : h^2EQ	.000	.000	.000	.001	.001	.005
ICC : h^2 : h^2EQ	.000	.000	.002	.004	.002	.006
ICCEQ : h^2 : h2EQ	.000	.000	.000	.000	.000	.001
N : ICC : CorB	.000	.000	.000	.003	.004	.017
N : ICCEQ : CorB	.000	.000	.000	.000	.000	.001
ICC : ICCEQ : CorB	.000	.000	.000	.001	.000	.005
N : h^2 : CorB	.000	.000	.000	.002	.001	.018
ICC : h^2 : CorB	.000	.000	.000	.004	.008	.009
ICCEQ : h^2 : CorB	.000	.000	.000	.001	.000	.000
N : h^2EQ : CorB	.000	.000	.000	.001	.000	.008
ICC : h^2EQ : CorB	.000	.000	.000	.002	.000	.007
ICCEQ : h^2EQ : CorB	.000	.000	.000	.000	.000	.001
h^2 : h^2EQ : CorB	.000	.000	.000	.001	.000	.005
N : ICC : Method	.013	.001	.001	.006	.006	.004
N : ICCEQ : Method	.000	.000	.000	.000	.000	.000
ICC : ICCEQ : Method	.000	.000	.002	.001	.000	.000
N : h^2 : Method	.001	.000	.001	.002	.000	.000
ICC : h^2 : Method	.001	.001	**.168**	**.143**	.001	.042
ICCEQ : h^2 : Method	.000	.000	.000	.000	.000	.000
N : h^2EQ : Method	.000	.000	.000	.000	.000	.000
ICC : h^2EQ : Method	.000	.000	.001	.001	.000	.000
ICCEQ : h^2EQ : Method	.000	.000	.000	.000	.000	.000
h^2 : h^2EQ : Method	.000	.000	.001	.001	.000	.000
N : CorB : Method	.000	.000	.000	.000	.003	.000
ICC : CorB : Method	.000	.000	.000	.004	**.145**	.010
ICCEQ : CorB : Method	.000	.000	.000	.000	.000	.000
h^2 : CorB : Method	.000	.000	.000	.000	.021	.002
h^2EQ : CorB : Method	.000	.000	.000	.000	.000	.000
N : ICC : ICCEQ : h^2	.000	.000	.000	.002	.000	.030
N : ICC : ICCEQ : h^2EQ	.000	.000	.000	.000	.000	.003
N : ICC : h^2 : h^2EQ	.000	.000	.001	.016	.005	.024
N : ICCEQ : h^2 : h^2EQ	.000	.000	.000	.000	.000	.001
ICC : ICCEQ : h^2 : h^2EQ	.000	.000	.000	.000	.000	.005
N : ICC : ICCEQ : CorB	.000	.000	.000	.003	.000	.009
N : ICC : h^2 : CorB	.000	.000	.000	.008	.007	.021
N : ICCEQ : h^2 : CorB	.000	.000	.000	.004	.000	.003
ICC : ICCEQ : h^2 : CorB	.000	.000	.000	.002	.000	.002
N : ICC : h^2EQ : CorB	.000	.000	.000	.005	.000	.021
N : ICCEQ : h^2EQ : CorB	.000	.000	.000	.000	.000	.001
ICC : ICCEQ : h^2EQ : CorB	.000	.000	.000	.000	.000	.003
N : h^2 : h^2EQ : CorB	.000	.000	.000	.002	.000	.018
ICC : h^2 : h^2EQ : CorB	.000	.000	.000	.001	.000	.018

ICCEQ : h^2 : h^2EQ : CorB	.000	.000	.000	.000	.000	.001
N : ICC : ICCEQ : Method	.000	.000	.000	.000	.000	.000
N : ICC : h^2 : Method	.002	.000	.000	.001	.001	.001
N : ICCEQ : h2 : Method	.000	.000	.000	.000	.000	.000
ICC : ICCEQ : h2 : Method	.000	.000	.000	.000	.000	.000
N : ICC : h^2EQ : Method	.000	.000	.000	.000	.000	.000
N : ICCEQ : h^2EQ : Method	.000	.000	.000	.000	.000	.000
ICC : ICCEQ : h^2EQ : Method	.000	.000	.000	.000	.000	.000
N : h^2 : h^2EQ : Method	.000	.000	.000	.000	.000	.000
ICC : h^2 : h^2EQ : Method	.000	.000	.000	.000	.000	.000
ICCEQ : h^2 : h^2EQ : Method	.000	.000	.000	.000	.000	.000
N : ICC : CorB : Method	.000	.000	.000	.000	.003	.000
N : ICCEQ : CorB : Method	.000	.000	.000	.000	.000	.000
ICC : ICCEQ : CorB : Method	.000	.000	.000	.000	.000	.000
N : h^2 : CorB : Method	.000	.000	.000	.000	.000	.000
ICC : h^2 : CorB : Method	.000	.000	.000	.002	.005	.001
ICCEQ : h^2 : CorB : Method	.000	.000	.000	.000	.000	.000
N : h^2EQ : CorB : Method	.000	.000	.000	.000	.000	.000
ICC : h^2EQ : CorB : Method	.000	.000	.000	.000	.000	.000
ICCEQ : h^2EQ : CorB : Method	.000	.000	.000	.000	.000	.000
h^2 : h^2EQ : CorB : Method	.000	.000	.000	.000	.000	.000
N : ICC : ICCEQ : h^2 : h^2EQ	.000	.000	.000	.001	.000	.022
N : ICC : ICCEQ : h^2 : CorB	.000	.000	.000	.004	.000	.020
N : ICC : ICCEQ : h^2EQ : CorB	.000	.000	.000	.001	.000	.009
N : ICC : h^2 : h^2EQ : CorB	.000	.000	.001	.004	.001	.028
N : ICCEQ : h^2 : h^2EQ : CorB	.000	.000	.000	.001	.000	.018
ICC : ICCEQ : h^2 : h^2EQ : CorB	.000	.000	.000	.001	.000	.004
N : ICC : ICCEQ : h^2 : Method	.000	.000	.000	.000	.000	.000
N : ICC : ICCEQ : h^2EQ : Method	.000	.000	.000	.000	.000	.000
N : ICC : h^2 : h^2EQ : Method	.000	.000	.000	.001	.000	.000
N : ICCEQ : h^2 : h^2EQ : Method	.000	.000	.000	.000	.000	.000
ICC : ICCEQ : h^2 : h^2EQ : Method	.000	.000	.000	.000	.000	.000
N : ICC : ICCEQ : CorB : Method	.000	.000	.000	.000	.000	.000
N : ICC : h^2 : CorB : Method	.000	.000	.001	.000	.001	.001
N : ICCEQ : h^2 : CorB : Method	.000	.000	.000	.000	.000	.000
ICC : ICCEQ : h^2 : CorB : Method	.000	.000	.000	.000	.000	.000
N : ICC : h^2EQ : CorB : Method	.000	.000	.000	.000	.000	.000
N : ICCEQ : h^2EQ : CorB : Method	.000	.000	.000	.000	.000	.000
ICC : ICCEQ : h^2EQ : CorB : Method	.000	.000	.000	.000	.000	.000
N : h^2 : h^2EQ : CorB : Method	.000	.000	.000	.000	.000	.000
ICC : h^2 : h^2EQ : CorB : Method	.000	.000	.000	.000	.000	.000
ICCEQ : h2 : h^2EQ : CorB : Method	.000	.000	.000	.000	.000	.000
N : ICC : ICCEQ : h^2 : h^2EQ : CorB	.000	.000	.000	.001	.002	.009
N : ICC : ICCEQ : h^2 : h^2EQ : Method	.000	.000	.000	.000	.000	.000

N : ICC : ICCEQ : h^2 : CorB : Method	.000	.000	.000	.000	.000	.000
N : ICC : ICCEQ : h^2EQ : CorB : Method	.000	.000	.000	.000	.000	.000
N : ICC : h^2 : h^2EQ : CorB : Method	.000	.000	.000	.000	.000	.000
N : ICCEQ : h^2 : h^2EQ : CorB : Method	.000	.000	.000	.000	.000	.000
ICC : ICCEQ : h^2 : h^2EQ : CorB : Method	.000	.000	.000	.000	.000	.000
Seven-way Interaction and Error	.000	.000	.000	.000	.000	.000

Note.

1. The bold values indicate η^2 greater than .05.
2. N = Sample size (100/5, 10/50, 400/20, and 40/200 where the first value is the number of clusters and the second value is cluster size), ICC = Average intraclass correlations across indicators (.05, .15, .25, .50, and .75), ICCEQ = Equal ICC across items within the same factor (Equal and Unequal), h^2 = Average macro-level communalities (low, medium, and high), h^2EQ = Equal macro-level communalities across items within the same factor (Equal and Unequal), CorB = Average macro-level factor correlation (.2, .5, and .8), and Method = The method of analysis (two-level MSEM, saturated-macro-level multilevel MSEM, and ignore-macro-level SEM)
3. RD = Relative Difference, RR = Rejection rate based on chi-square test, RMSEA = Root mean square error of approximation, SE = Standard error, CorW = The estimated micro-level correlation

Table D1

The full results of Eta-squared values for analysis of variance table of the simulation conditions for rejection rate (%sig), root mean square error of approximation (RMSEA), estimated macro-level standardized loadings, bias in standard error of macro-level standardized loadings, estimated macro-level factor correlation (CorB), and bias in standard error of macro-level factor correlation.

Effects	RR	RMSEA	Loading	RB SE Loading	CorB	RB SE CorB
N	**.215**	**.283**	.007	.038	**.062**	**.055**
ICC	**.149**	.010	.008	.007	**.088**	.018
ICCEQ	.000	.000	.001	.000	.000	.001
h^2	.002	.000	**.144**	.013	.001	.010
h^2EQ	.001	.000	.000	.000	.000	.000
CorW	.001	.000	.000	.000	**.093**	.002
Method	**.087**	**.559**	.023	**.179**	.023	**.144**
N : ICC	**.150**	.013	.033	**.145**	**.081**	**.199**
N : ICCEQ	.000	.000	.000	.001	.000	.001
ICC : ICCEQ	.001	.000	.000	.001	.001	.003
N : h^2	.003	.000	.025	.006	.000	.004
ICC : h^2	.010	.001	**.232**	.024	.002	.013
ICCEQ : h^2	.001	.000	.000	.000	.000	.000
N : h^2EQ	.000	.000	.000	.000	.000	.000
ICC : h^2EQ	.000	.000	.001	.000	.000	.000
ICCEQ : h^2EQ	.001	.000	.000	.000	.000	.000
h^2 : h^2EQ	.000	.000	.000	.000	.000	.000
N : CorW	.002	.000	.000	.000	.016	.002
ICC : CorW	.001	.000	.000	.001	**.143**	.005
ICCEQ : CorW	.000	.000	.000	.000	.000	.000
h^2 : CorW	.000	.000	.000	.000	.009	.000
h^2EQ : CorW	.000	.000	.000	.000	.000	.000
N : Method	.037	**.096**	.014	**.059**	.028	.049
ICC : Method	**.127**	.015	.044	**.347**	**.070**	**.293**
ICCEQ : Method	.007	.000	.001	.001	.000	.000
h^2 : Method	.007	.000	**.144**	.001	.000	.006
h^2EQ : Method	.000	.000	.000	.000	.000	.000
CorW : Method	.001	.000	.000	.000	**.094**	.001
N : ICC : ICCEQ	.003	.000	.000	.002	.000	.004
N : ICC : h^2	.003	.001	.029	.013	.001	.008
N : ICCEQ : h^2	.003	.000	.000	.000	.000	.000
ICC : ICCEQ : h^2	.003	.000	.000	.000	.000	.000
N : ICC : h^2EQ	.002	.000	.000	.001	.000	.000
N : ICCEQ : h^2EQ	.000	.000	.000	.000	.000	.000
ICC : ICCEQ : h^2EQ	.000	.000	.000	.000	.000	.000
N : h^2 : h^2EQ	.001	.000	.000	.000	.000	.000
ICC : h^2 : h^2EQ	.000	.000	.000	.000	.000	.000
ICCEQ : h^2 : h2EQ	.000	.000	.000	.000	.000	.000
N : ICC : CorW	.001	.000	.000	.001	.017	.004
N : ICCEQ : CorW	.001	.000	.000	.000	.000	.000
ICC : ICCEQ : CorW	.000	.000	.000	.001	.000	.000

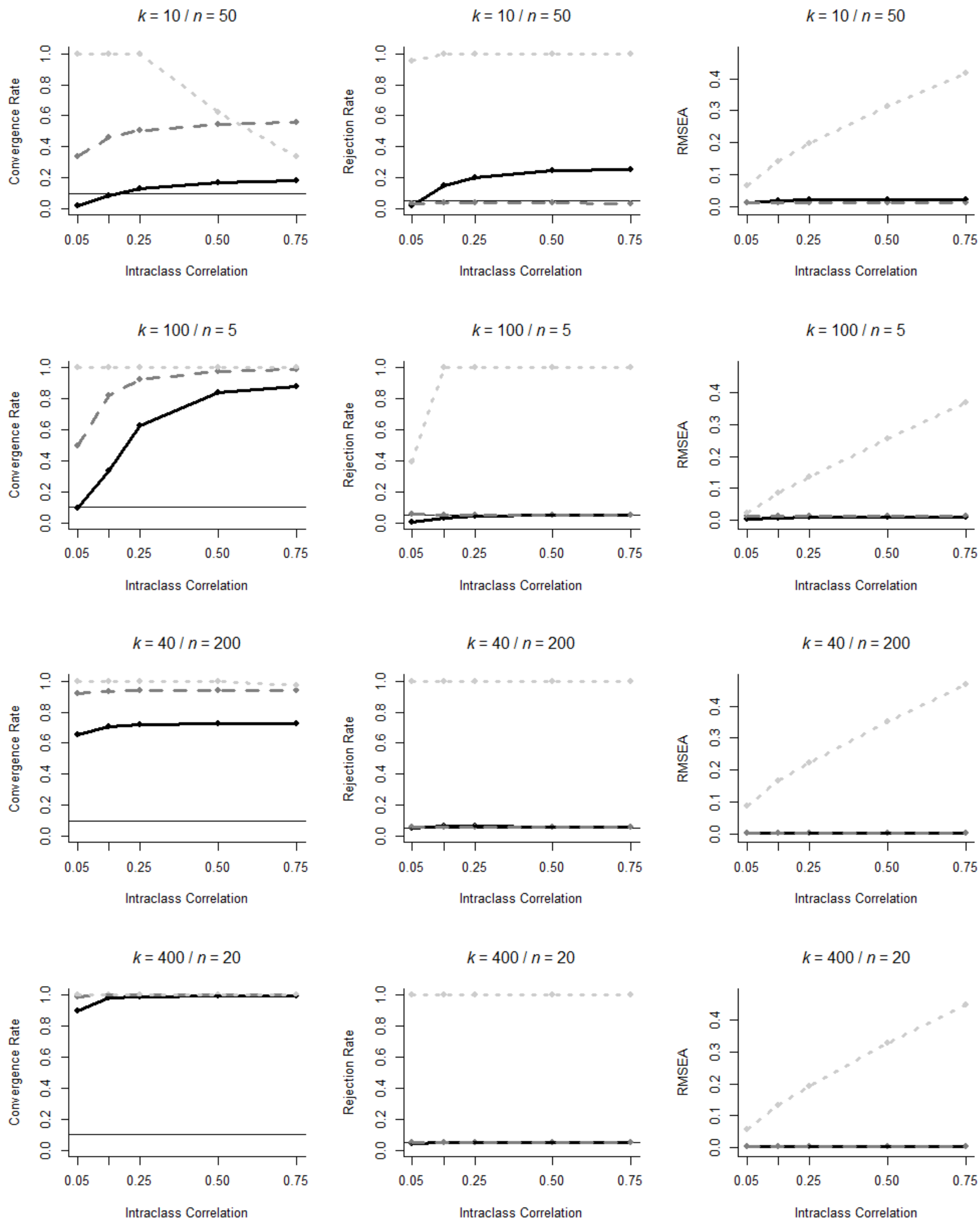
N : h^2 : CorW	.003	.000	.000	.000	.001	.000
ICC : h^2 : CorW	.000	.000	.000	.001	.009	.003
ICCEQ : h^2 : CorW	.001	.000	.000	.000	.000	.000
N : h^2EQ : CorW	.000	.000	.000	.000	.000	.000
ICC : h^2EQ : CorW	.000	.000	.000	.000	.000	.000
ICCEQ : h^2EQ : CorW	.000	.000	.000	.000	.000	.000
h^2 : h^2EQ : CorW	.000	.000	.000	.000	.000	.000
N : ICC : Method	.049	.016	.032	**.120**	**.071**	**.111**
N : ICCEQ : Method	.000	.000	.000	.000	.000	.000
ICC : ICCEQ : Method	.003	.000	.000	.002	.001	.001
N : h^2 : Method	.007	.000	.022	.001	.000	.000
ICC : h^2 : Method	.005	.000	**.213**	.001	.000	.006
ICCEQ : h^2 : Method	.008	.000	.000	.000	.000	.000
N : h^2EQ : Method	.001	.000	.000	.000	.000	.000
ICC : h^2EQ : Method	.001	.000	.001	.000	.000	.000
ICCEQ : h^2EQ : Method	.000	.000	.000	.000	.000	.000
h^2 : h^2EQ : Method	.001	.000	.000	.000	.000	.000
N : CorW : Method	.004	.000	.000	.000	.015	.001
ICC : CorW : Method	.003	.000	.000	.000	**.132**	.003
ICCEQ : CorW : Method	.000	.000	.000	.000	.000	.000
h^2 : CorW : Method	.001	.000	.000	.000	.008	.000
h^2EQ : CorW : Method	.000	.000	.000	.000	.000	.000
N : ICC : ICCEQ : h^2	.004	.000	.000	.000	.000	.001
N : ICC : ICCEQ : h^2EQ	.001	.000	.000	.000	.000	.001
N : ICC : h^2 : h^2EQ	.000	.000	.000	.001	.000	.000
N : ICCEQ : h^2 : h^2EQ	.001	.000	.000	.000	.000	.000
ICC : ICCEQ : h^2 : h^2EQ	.000	.000	.000	.000	.000	.000
N : ICC : ICCEQ : CorW	.001	.000	.000	.002	.000	.001
N : ICC : h^2 : CorW	.002	.000	.000	.001	.000	.004
N : ICCEQ : h^2 : CorW	.002	.000	.000	.000	.000	.001
ICC : ICCEQ : h^2 : CorW	.001	.000	.000	.000	.000	.002
N : ICC : h^2EQ : CorW	.001	.000	.000	.001	.000	.000
N : ICCEQ : h^2EQ : CorW	.000	.000	.000	.000	.000	.000
ICC : ICCEQ : h^2EQ : CorW	.000	.000	.000	.000	.000	.001
N : h^2 : h^2EQ : CorW	.001	.000	.000	.000	.000	.000
ICC : h^2 : h^2EQ : CorW	.000	.000	.000	.000	.000	.000
ICCEQ : h^2 : h^2EQ : CorW	.000	.000	.000	.000	.000	.000
N : ICC : ICCEQ : Method	.005	.000	.000	.001	.000	.002
N : ICC : h^2 : Method	.006	.000	.020	.005	.001	.002
N : ICCEQ : h2 : Method	.004	.000	.000	.000	.000	.000
ICC : ICCEQ : h2 : Method	.005	.000	.000	.000	.000	.000
N : ICC : h^2EQ : Method	.001	.000	.000	.000	.000	.000
N : ICCEQ : h^2EQ : Method	.000	.000	.000	.000	.000	.000
ICC : ICCEQ : h^2EQ : Method	.001	.000	.000	.000	.000	.000
N : h^2 : h^2EQ : Method	.000	.000	.000	.000	.000	.000
ICC : h^2 : h^2EQ : Method	.000	.000	.000	.000	.000	.000
ICCEQ : h^2 : h^2EQ : Method	.000	.000	.000	.000	.000	.000
N : ICC : CorW : Method	.004	.000	.000	.001	.014	.006

N : ICCEQ : CorW : Method	.002	.000	.000	.000	.000	.000
ICC : ICCEQ : CorW : Method	.001	.000	.000	.001	.000	.000
N : h^2 : CorW : Method	.004	.000	.000	.000	.001	.000
ICC : h^2 : CorW : Method	.001	.000	.000	.000	.009	.001
ICCEQ : h^2 : CorW : Method	.000	.000	.000	.000	.000	.000
N : h^2EQ : CorW : Method	.000	.000	.000	.000	.000	.000
ICC : h^2EQ : CorW : Method	.000	.000	.000	.000	.000	.000
ICCEQ : h^2EQ : CorW : Method	.000	.000	.000	.000	.000	.000
h^2 : h^2EQ : CorW : Method	.000	.000	.000	.000	.000	.000
N : ICC : ICCEQ : h^2 : h^2EQ	.001	.000	.000	.000	.000	.000
N : ICC : ICCEQ : h^2 : CorW	.001	.000	.000	.000	.000	.005
N : ICC : ICCEQ : h^2EQ : CorW	.001	.000	.000	.001	.000	.001
N : ICC : h^2 : h^2EQ : CorW	.001	.000	.000	.001	.000	.000
N : ICCEQ : h^2 : h^2EQ : CorW	.000	.000	.000	.000	.000	.001
ICC : ICCEQ : h^2 : h^2EQ : CorW	.000	.000	.000	.000	.000	.001
N : ICC : ICCEQ : h^2 : Method	.008	.000	.000	.000	.000	.001
N : ICC : ICCEQ : h^2EQ : Method	.001	.000	.000	.000	.000	.000
N : ICC : h^2 : h^2EQ : Method	.002	.000	.000	.000	.000	.000
N : ICCEQ : h^2 : h^2EQ : Method	.000	.000	.000	.000	.000	.000
ICC : ICCEQ : h^2 : h^2EQ : Method	.000	.000	.000	.000	.000	.000
N : ICC : ICCEQ : CorW : Method	.003	.000	.000	.002	.000	.001
N : ICC : h^2 : CorW : Method	.004	.000	.000	.001	.001	.001
N : ICCEQ : h^2 : CorW : Method	.004	.000	.000	.000	.000	.001
ICC : ICCEQ : h^2 : CorW : Method	.001	.000	.000	.000	.000	.001
N : ICC : h^2EQ : CorW : Method	.002	.000	.000	.001	.000	.000
N : ICCEQ : h^2EQ : CorW : Method	.000	.000	.000	.000	.000	.000
ICC : ICCEQ : h^2EQ : CorW : Method	.000	.000	.000	.000	.000	.000
N : h^2 : h^2EQ : CorW : Method	.001	.000	.000	.000	.000	.000
ICC : h^2 : h^2EQ : CorW : Method	.001	.000	.000	.000	.000	.000
ICCEQ : h2 : h^2EQ : CorW : Method	.000	.000	.000	.000	.000	.000
N : ICC : ICCEQ : h^2 : h^2EQ : CorW	.001	.000	.000	.001	.000	.002
N : ICC : ICCEQ : h^2 : h^2EQ : Method	.001	.000	.000	.000	.000	.000
N : ICC : ICCEQ : h^2 : CorW : Method	.006	.000	.000	.000	.000	.003
N : ICC : ICCEQ : h^2EQ : CorW : Method	.001	.000	.000	.001	.000	.001
N : ICC : h^2 : h^2EQ : CorW : Method	.002	.000	.000	.000	.000	.000
N : ICCEQ : h^2 : h^2EQ : CorW : Method	.000	.000	.000	.000	.000	.000
ICC : ICCEQ : h^2 : h^2EQ : CorW : Method	.000	.000	.000	.000	.000	.000
Seven-way Interaction and Error	.001	.000	.000	.001	.000	.001

Note.

1. The bold values indicate η^2 greater than .05.
2. N = Sample size (50/10, 50/40, 200/10, and 200/40 where the first value is the number of clusters and the second value is cluster size), ICC = Average intraclass correlations across indicators (.05, .25, .50, .75, and .95), ICCEQ = Equal ICC across items within the same factor (Equal and Unequal), h^2 = Average micro-level communalities (low, medium, and high), h^2EQ = Equal macro-level communalities across items within the same factor (Equal and Unequal), CorB = Average micro-level factor correlation (.2, .5, and .8), and Method = The method of analysis (two-level MSEM, saturated-micro-level multilevel MSEM, and ignore-micro-level SEM)

3. RB = Relative Bias, RR = Rejection rate based on chi-square test, RMSEA = Root mean square error of approximation, SE = Standard error, CorW = The estimated micro-level correlation


k = 10 / n = 50
k = 10 / n = 50
k = 10 / n = 50
k = 100 / n = 5
k = 100 / n = 5
k = 100 / n = 5
k = 40 / n = 200
k = 40 / n = 200
k = 40 / n = 200
k = 400 / n = 20
k = 400 / n = 20
k = 400 / n = 20
Convergence Rate
Rejection Rate
RMSEA
Intraclass Correlation
0.05
0.25
0.50
0.75
0.0
0.2
0.4
0.6
0.8
1.0
0.1
0.3

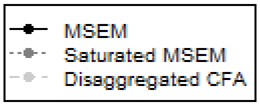


Figure C1. Convergence rate (left column), rejection rate based on the chi-square statistics (middle column), and the average RMSEA (right column) of Simulation Study 1. k is the number of clusters and n is the cluster size. The solid horizontal lines denote a convergence rate of .10 (left column) and a rejection rate of .05, the nominal alpha (middle column).

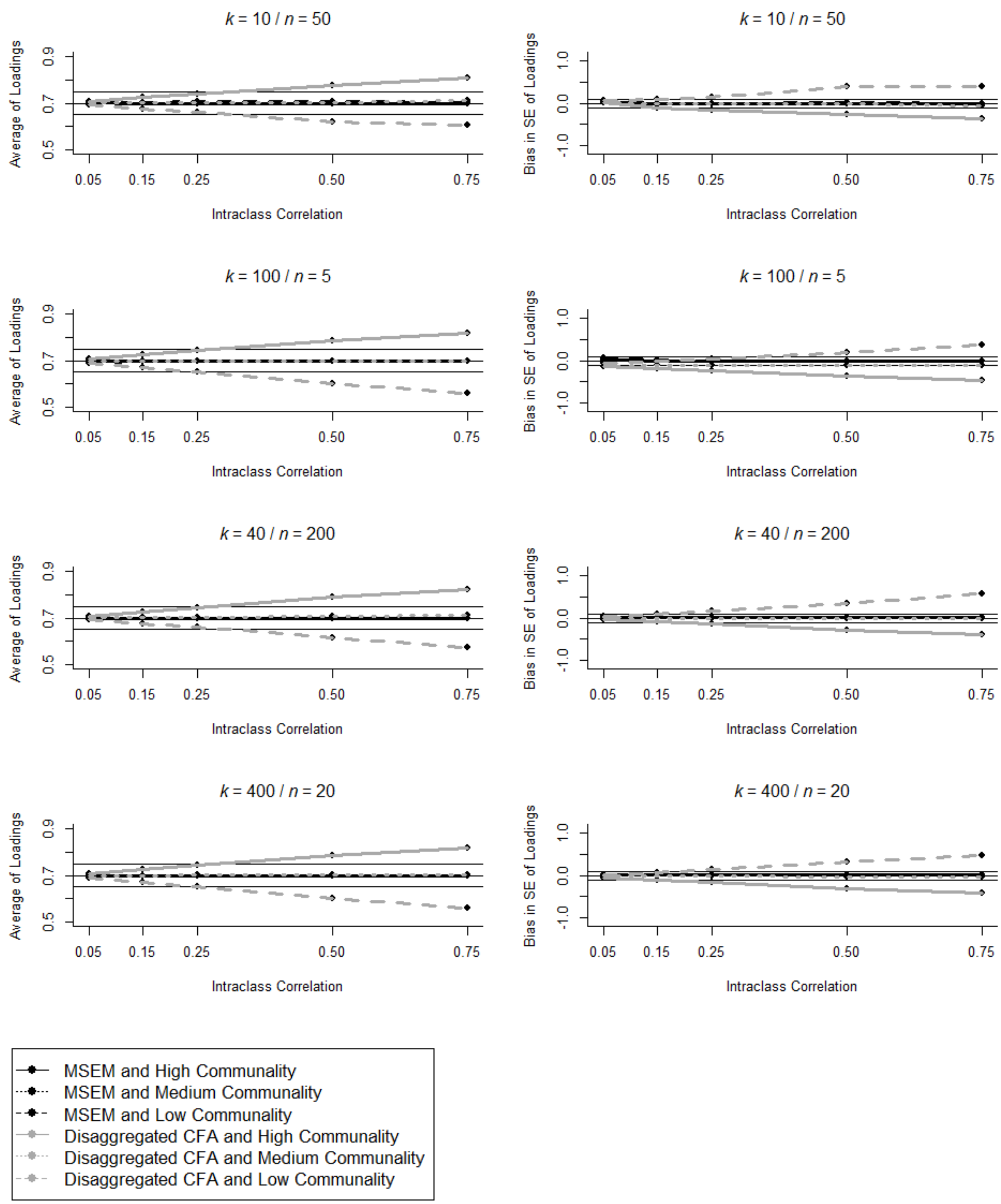


Figure C2. The average standardized factor loadings (left column) and the relative difference in standard errors of standardized factor loadings (right column) in each condition. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines in each plot denote absolute differences of -.05, 0, and .05 to represent the acceptable range of difference for the average standardized factor loadings (left column) and relative differences of -0.1, 0, and 0.1 to represent the acceptable range of difference for the relative difference in the standard errors (right column).

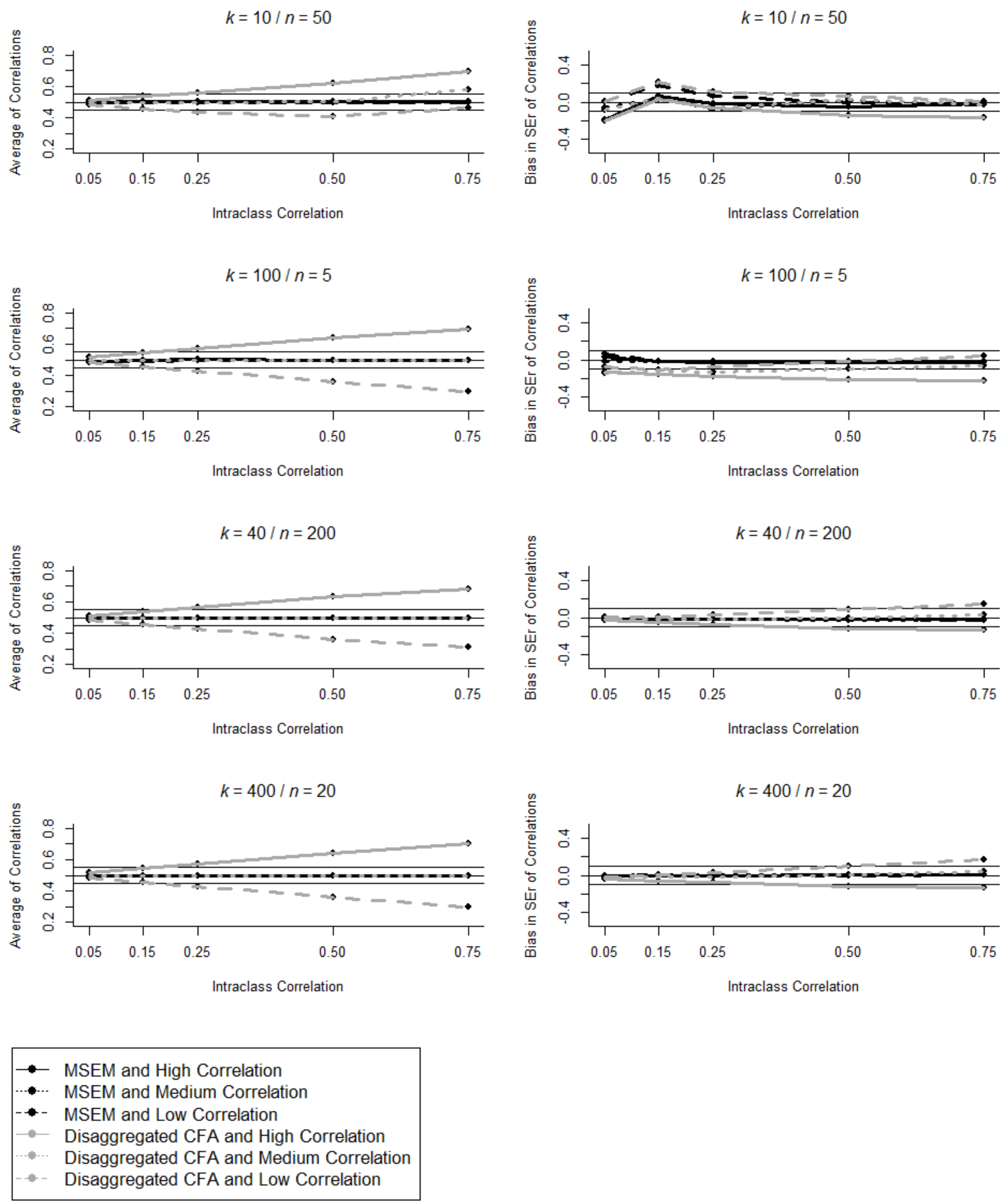
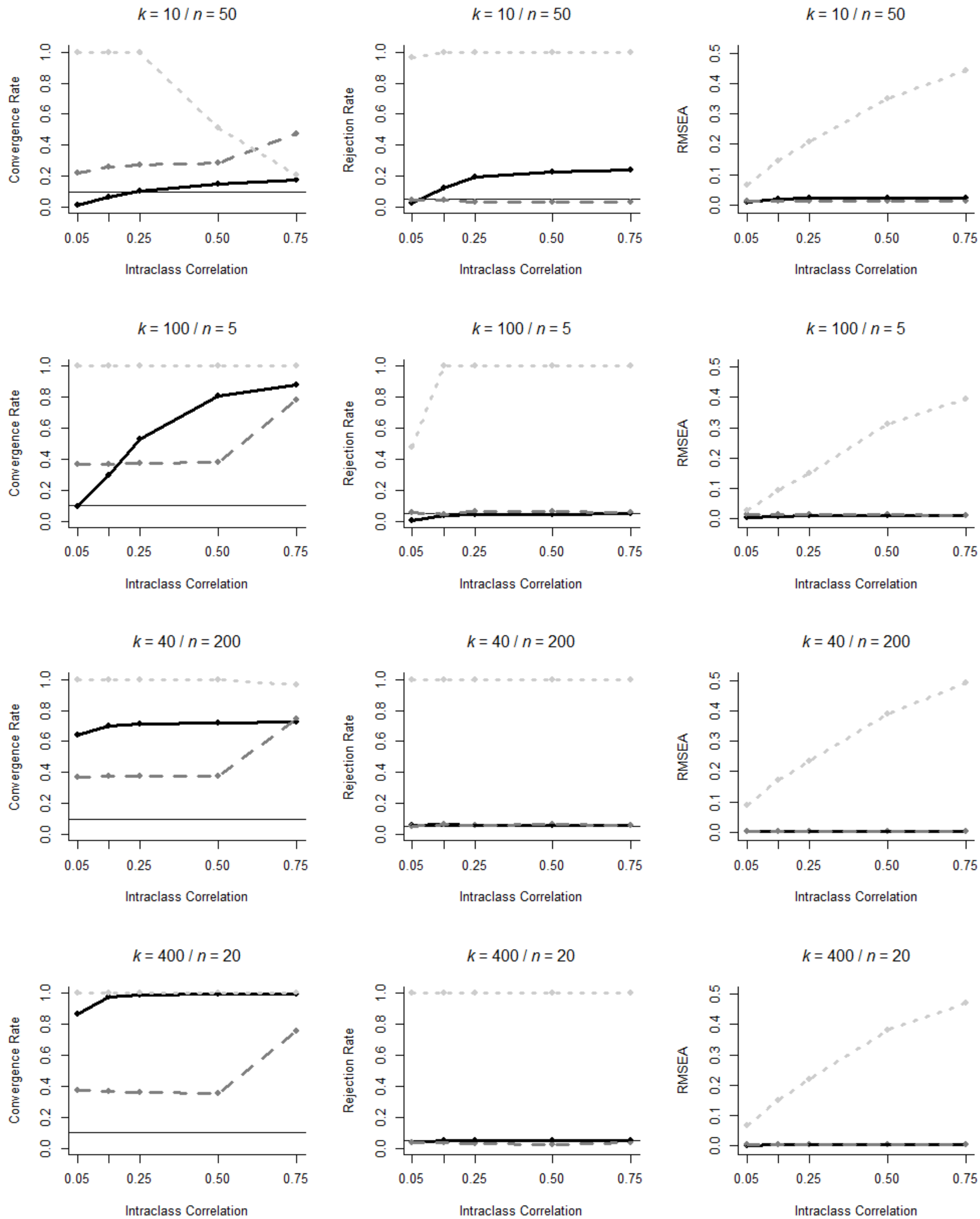


Figure C3. The average factor correlation (left column) and the relative difference in standard errors of factor correlation (right column) in each condition. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines in each plot denote absolute differences of -.05, 0, and .05 to represent the acceptable range of difference for the average factor correlation (left column) and relative differences of -0.1, 0, and 0.1 to represent the acceptable range of difference for the relative difference in the standard errors (right column).


k = 10 / n = 50
Convergence Rate
Intraclass Correlation
k = 10 / n = 50
Rejection Rate
Intraclass Correlation
k = 10 / n = 50
RMSEA
Intraclass Correlation
k = 100 / n = 5
Convergence Rate
Intraclass Correlation
k = 100 / n = 5
Rejection Rate
Intraclass Correlation
k = 100 / n = 5
RMSEA
Intraclass Correlation
k = 40 / n = 200
Convergence Rate
Intraclass Correlation
k = 40 / n = 200
Rejection Rate
Intraclass Correlation
k = 40 / n = 200
RMSEA
Intraclass Correlation
k = 400 / n = 20
Convergence Rate
Intraclass Correlation
k = 400 / n = 20
Rejection Rate
Intraclass Correlation
k = 400 / n = 20
RMSEA
Intraclass Correlation

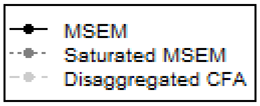


Figure C4. Convergence rate (left column), rejection rate based on the chi-square statistics (middle column), and the average RMSEA (right column) of the disaggregation simulation when the ICCs are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines denote a convergence rate of .10 (left column) and a rejection rate of .05, the nominal alpha (middle column).

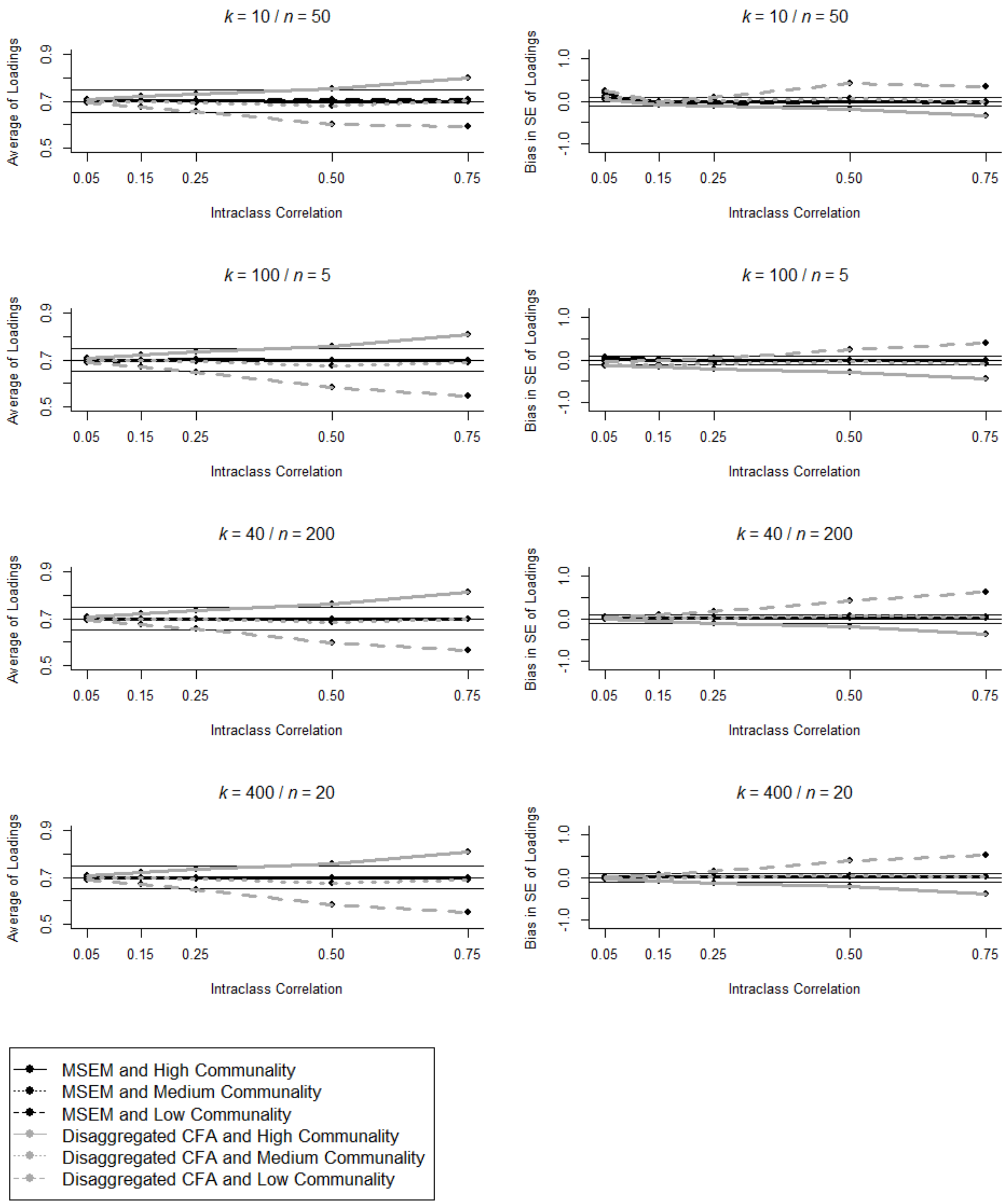


Figure C5. The average standardized factor loadings (left column) and the relative difference in standard errors of standardized factor loadings (right column) in each condition of the disaggregation simulation when the ICCs are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines in each plot denote absolute differences of -.05, 0, and .05 to represent the acceptable range of difference for the average standardized factor loadings (left column) and relative differences of -0.1, 0, and 0.1 to represent the acceptable range of difference for the relative difference in the standard errors (right column).

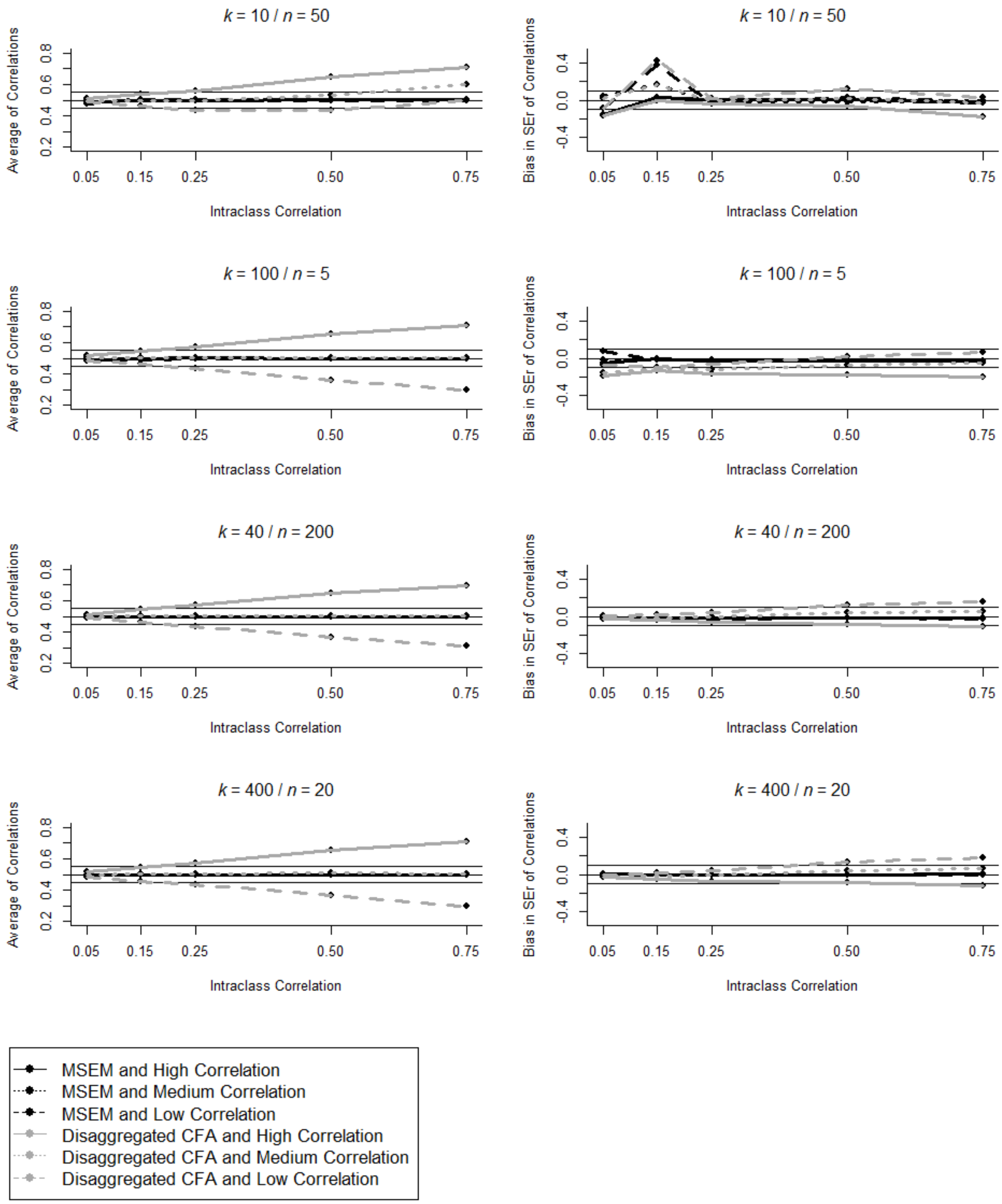
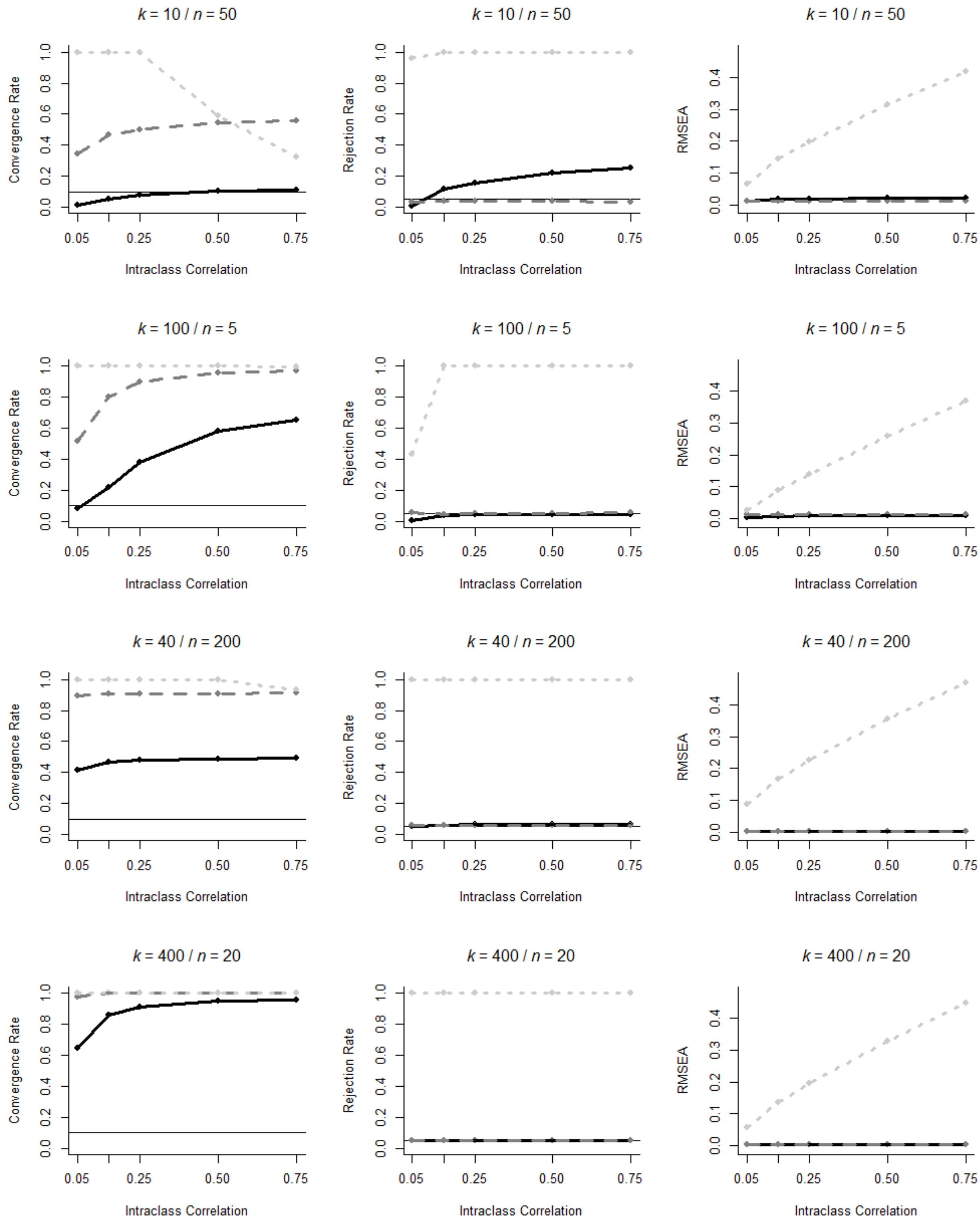


Figure C6. The average factor correlation (left column) and the relative difference in standard errors of factor correlation (right column) in each condition of the disaggregation simulation

when the ICCs are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines in each plot denote absolute differences of -.05, 0, and .05 to represent the acceptable range of difference for the average factor correlation (left column) and relative differences of -0.1, 0, and 0.1 to represent the acceptable range of difference for the relative difference in the standard errors (right column).


k = 10 / n = 50
Convergence Rate
Intraclass Correlation
k = 10 / n = 50
Rejection Rate
Intraclass Correlation
k = 10 / n = 50
RMSEA
Intraclass Correlation
k = 100 / n = 5
Convergence Rate
Intraclass Correlation
k = 100 / n = 5
Rejection Rate
Intraclass Correlation
k = 100 / n = 5
RMSEA
Intraclass Correlation
k = 40 / n = 200
Convergence Rate
Intraclass Correlation
k = 40 / n = 200
Rejection Rate
Intraclass Correlation
k = 40 / n = 200
RMSEA
Intraclass Correlation
k = 400 / n = 20
Convergence Rate
Intraclass Correlation
k = 400 / n = 20
Rejection Rate
Intraclass Correlation
k = 400 / n = 20
RMSEA
Intraclass Correlation

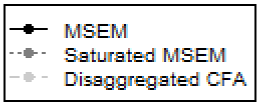


Figure C7. Convergence rate (left column), rejection rate based on the chi-square statistics (middle column), and the average RMSEA (right column) of the disaggregation simulation when the communalities are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines denote a convergence rate of .10 (left column) and a rejection rate of .05, the nominal alpha (middle column).

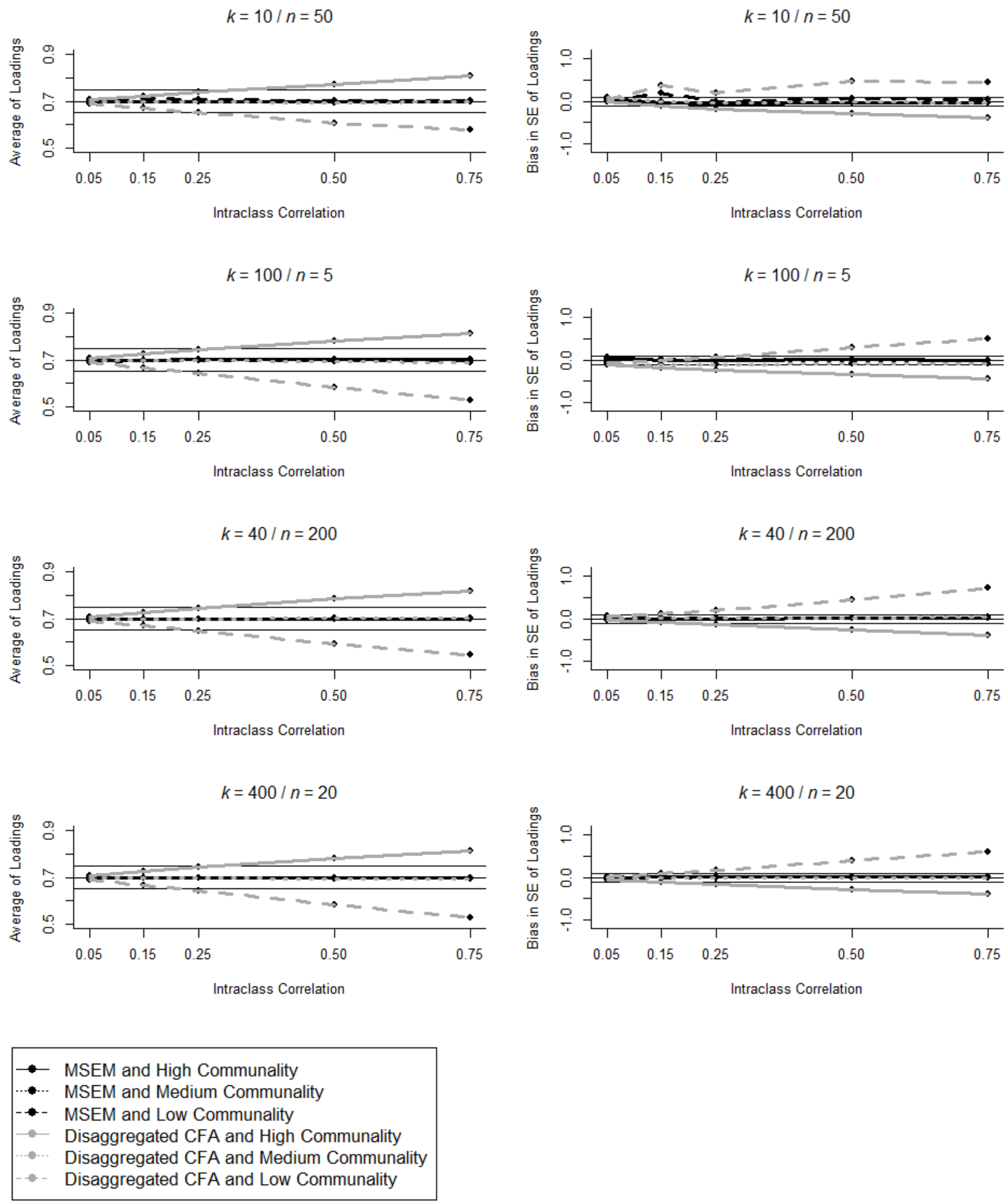


Figure C8. The average standardized factor loadings (left column) and the relative difference in standard errors of standardized factor loadings (right column) in each condition of the disaggregation simulation when the communalities are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines in each plot denote absolute differences of -.05, 0, and .05 to represent the acceptable range of difference for the average standardized factor loadings (left column) and relative differences of -0.1, 0, and 0.1 to represent the acceptable range of difference for the relative difference in the standard errors (right column).

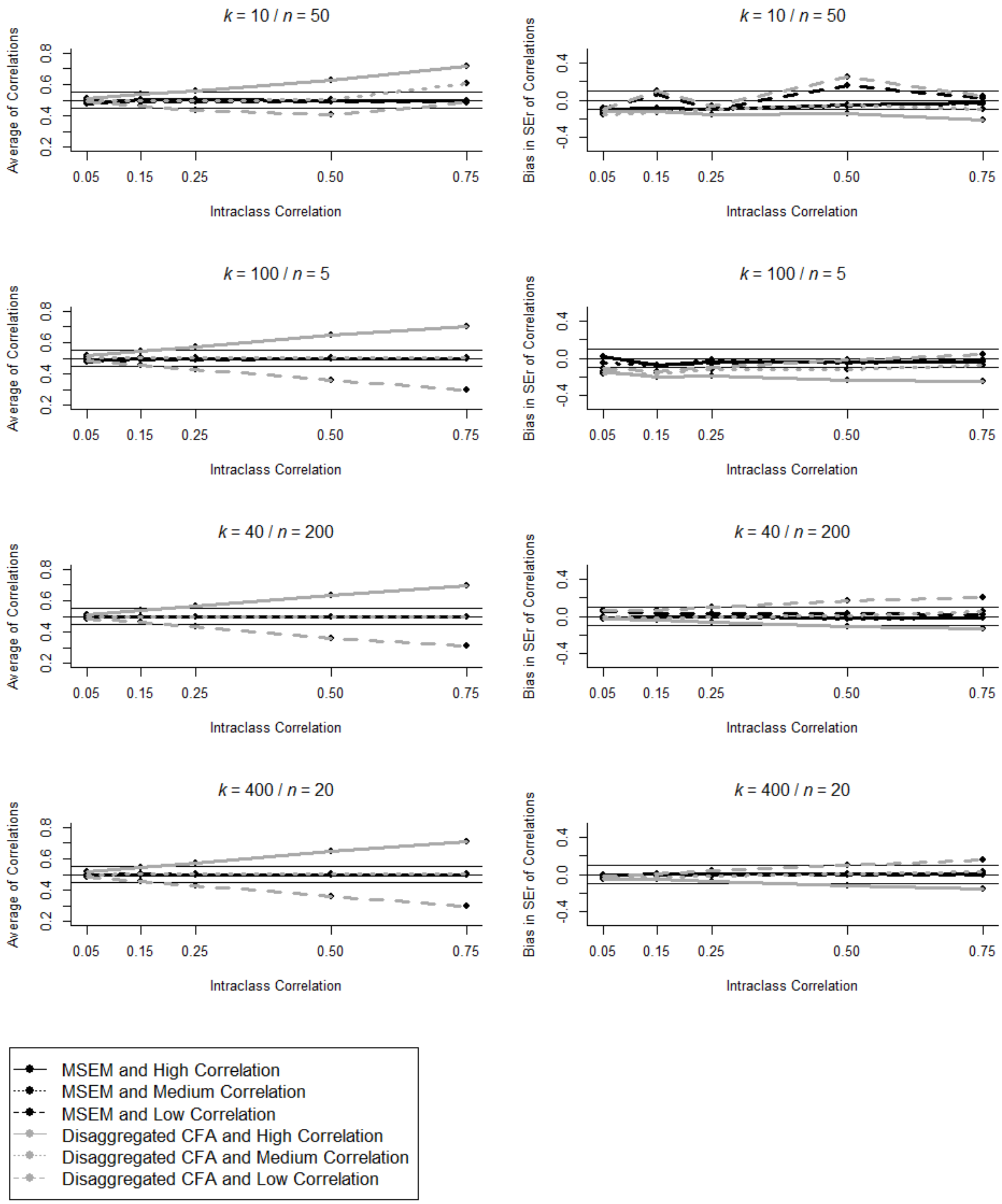
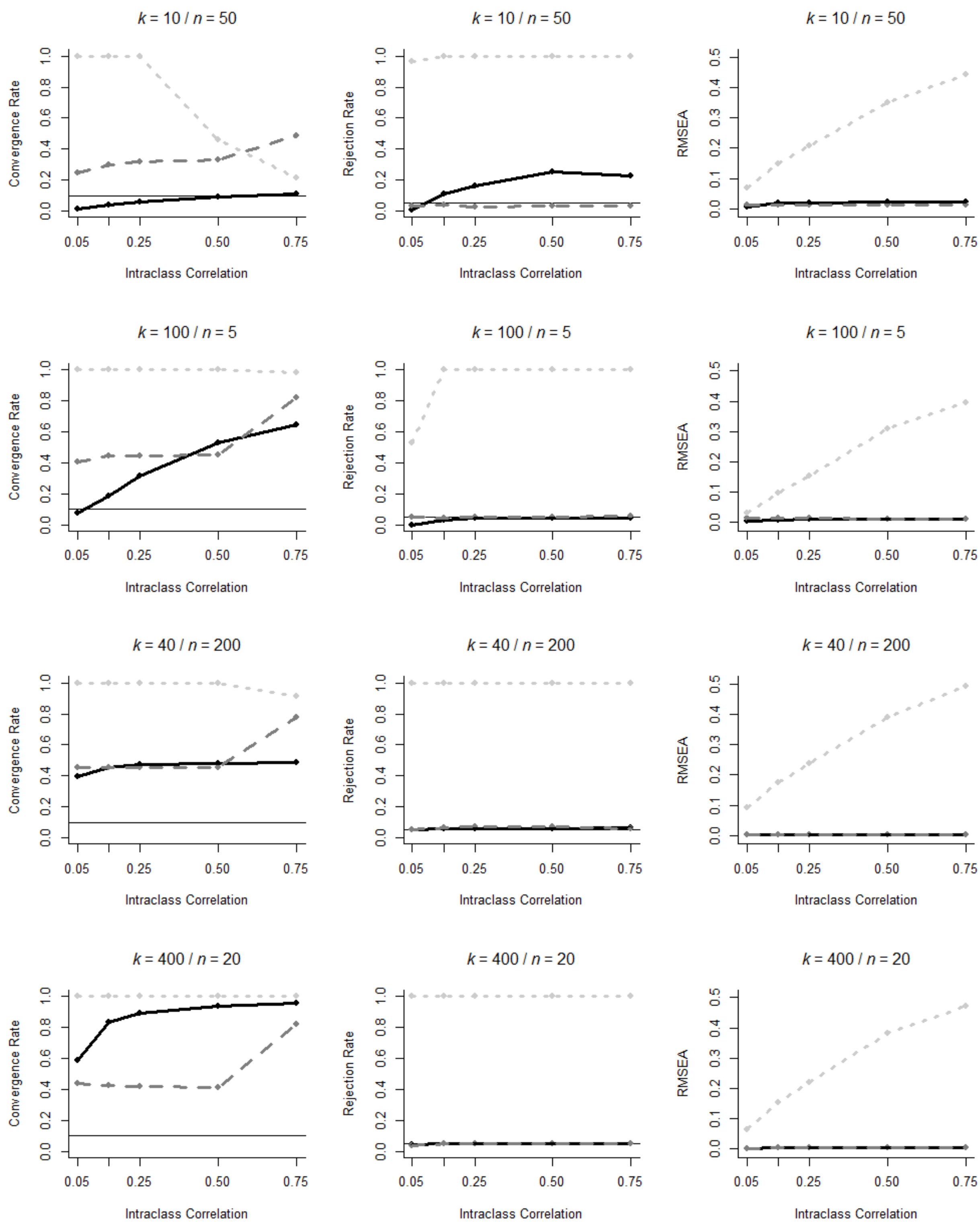


Figure C9. The average factor correlation (left column) and the relative difference in standard errors of factor correlation (right column) in each condition of the disaggregation simulation

when the communalities are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines in each plot denote absolute differences of -.05, 0, and .05 to represent the acceptable range of difference for the average factor correlation (left column) and relative differences of -0.1, 0, and 0.1 to represent the acceptable range of difference for the relative difference in the standard errors (right column).


k = 10 / n = 50
k = 10 / n = 50
k = 10 / n = 50
Convergence Rate
Rejection Rate
RMSEA
Intraclass Correlation
0.05 0.25 0.50 0.75
k = 100 / n = 5
k = 100 / n = 5
k = 100 / n = 5
k = 40 / n = 200
k = 40 / n = 200
k = 40 / n = 200
k = 400 / n = 20
k = 400 / n = 20
k = 400 / n = 20

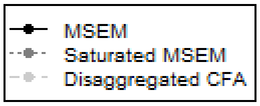


Figure C10. Convergence rate (left column), rejection rate based on the chi-square statistics (middle column), and the average RMSEA (right column) of the disaggregation simulation when the ICCs and the communalities are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines denote a convergence rate of .10 (left column) and a rejection rate of .05, the nominal alpha (middle column).

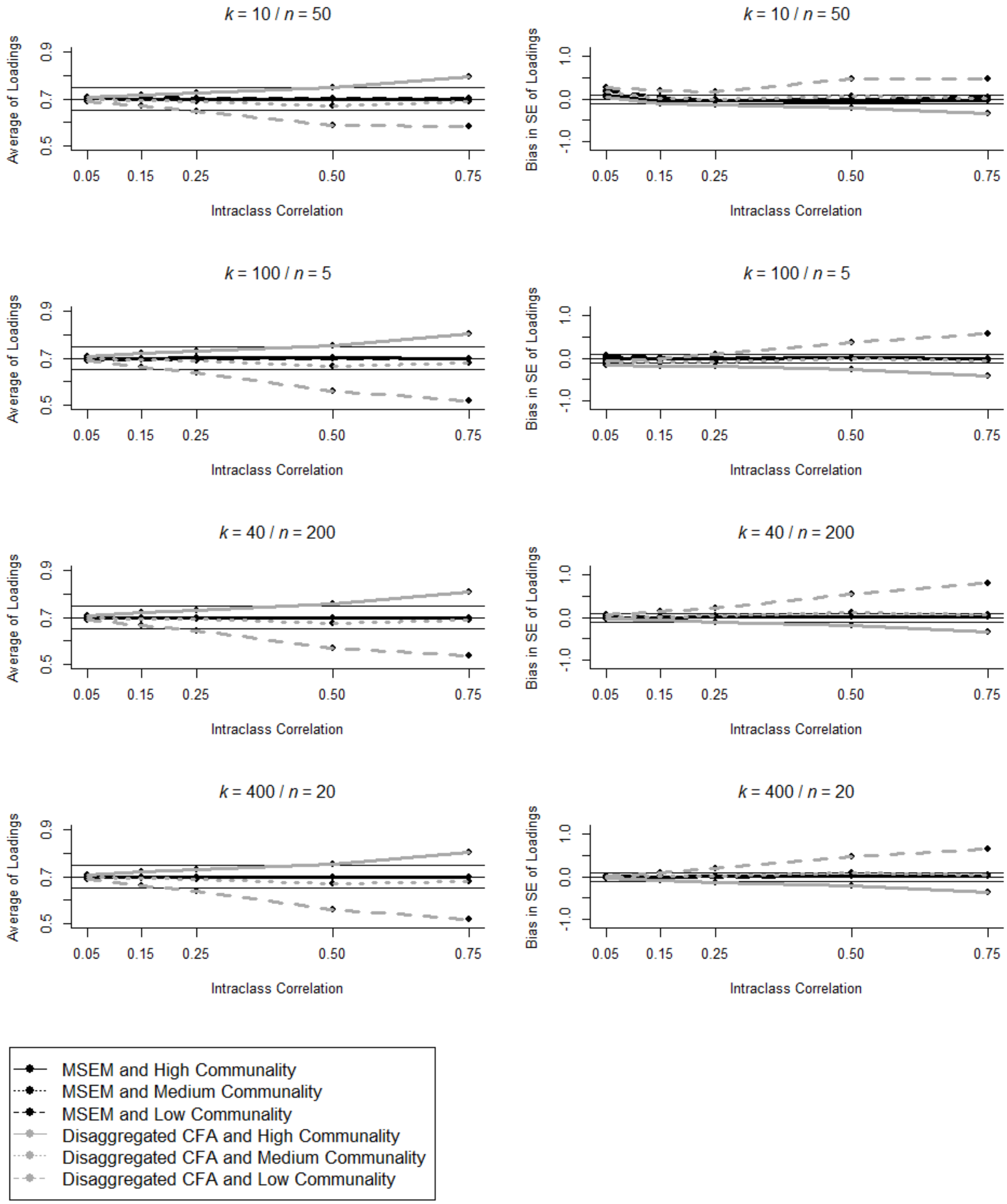


Figure C11. The average standardized factor loadings (left column) and the relative difference in standard errors of standardized factor loadings (right column) in each condition of the disaggregation simulation when the ICCs and the communalities are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines in each plot denote absolute differences of -.05, 0, and .05 to represent the acceptable range of difference for the average standardized factor loadings (left column) and relative differences of -0.1, 0, and 0.1 to represent the acceptable range of difference for the relative difference in the standard errors (right column).

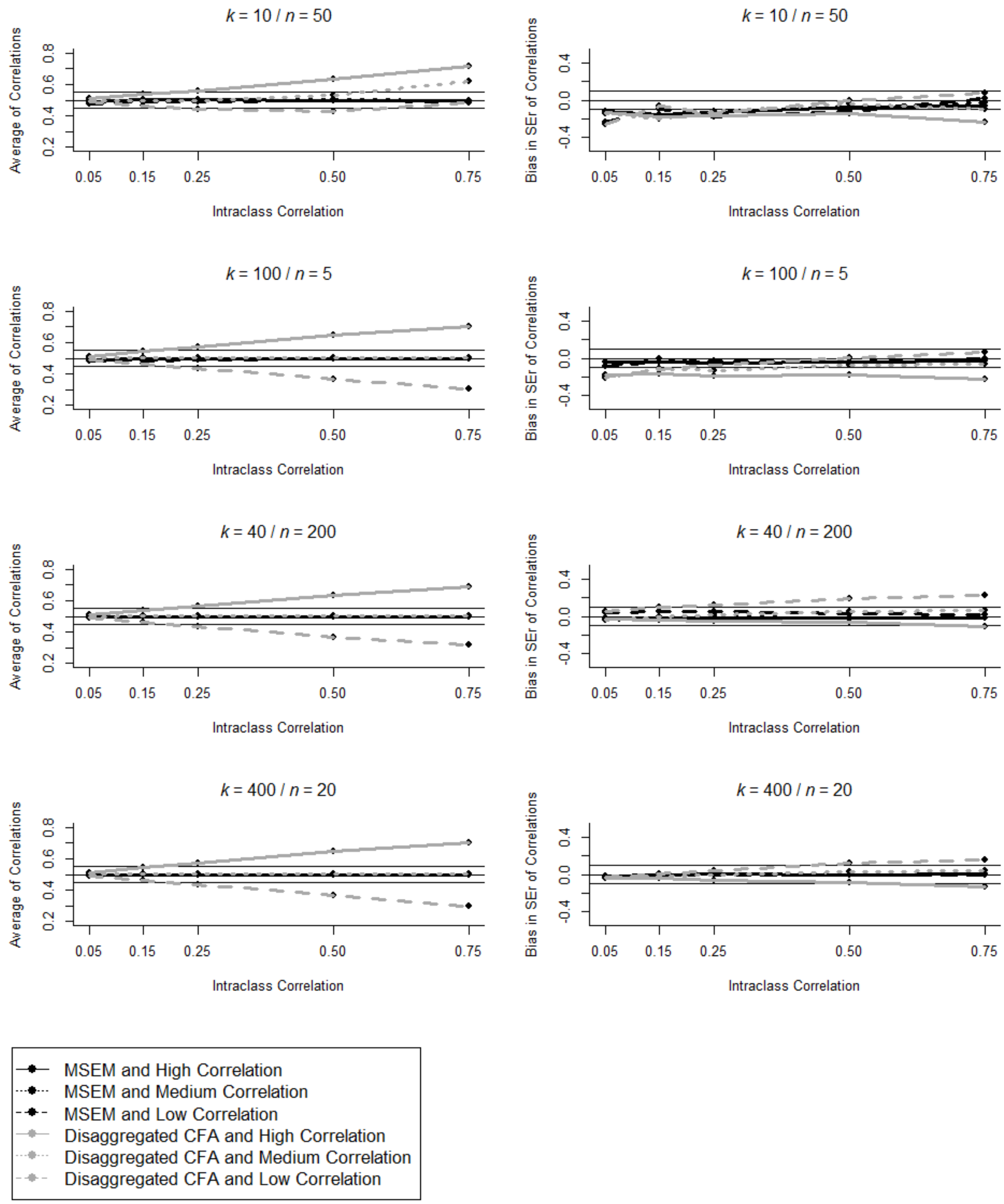
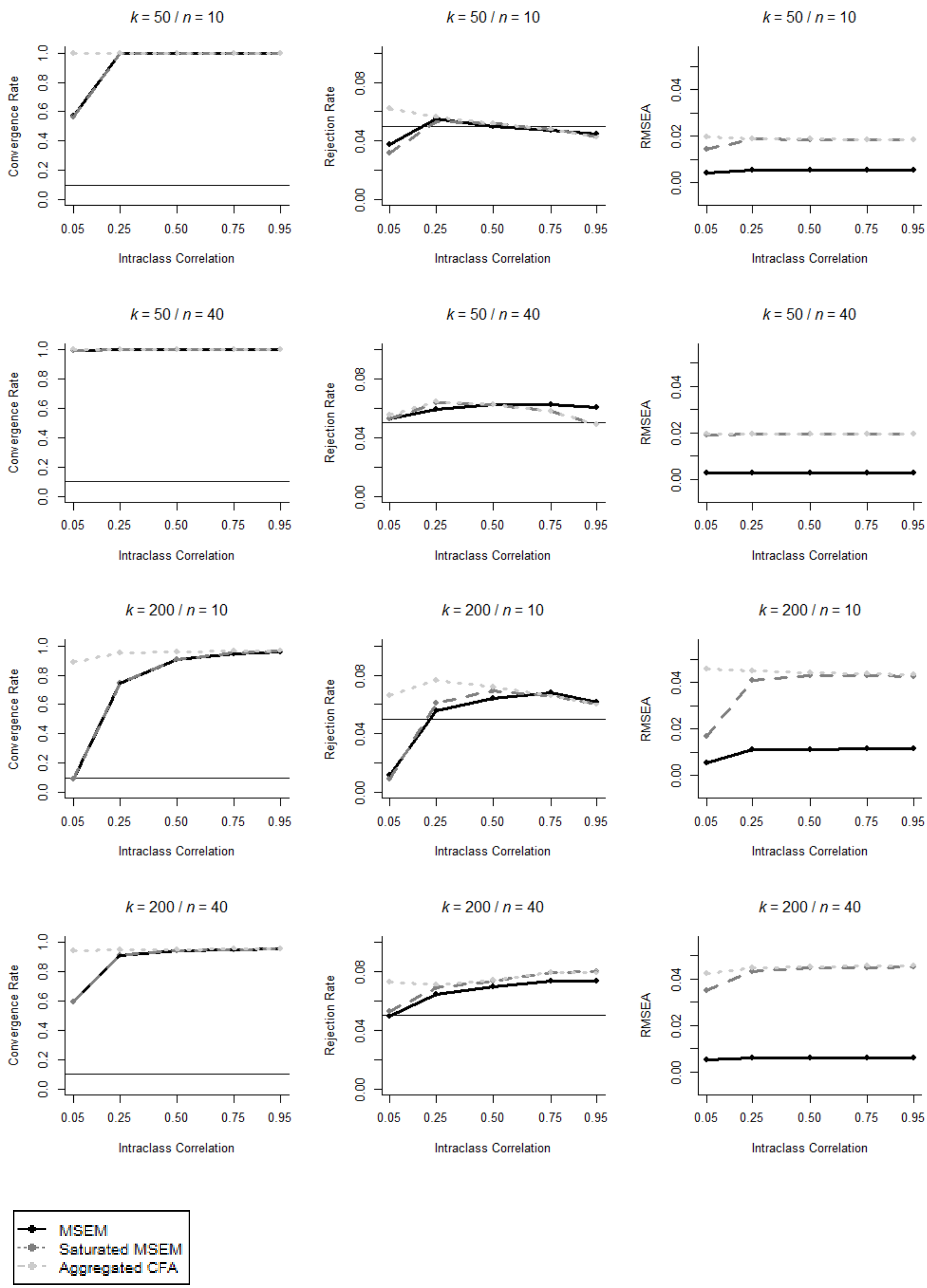


Figure C12. The average factor correlation (left column) and the relative difference in standard errors of factor correlation (right column) in each condition of the disaggregation simulation

when the ICCs and the communalities are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines in each plot denote absolute differences of -.05, 0, and .05 to represent the acceptable range of difference for the average factor correlation (left column) and relative differences of -0.1, 0, and 0.1 to represent the acceptable range of difference for the relative difference in the standard errors (right column).



k = 50 / n = 10
k = 50 / n = 10
k = 50 / n = 10
k = 50 / n = 40
k = 50 / n = 40
k = 50 / n = 40
k = 200 / n = 10
k = 200 / n = 10
k = 200 / n = 10
k = 200 / n = 40
k = 200 / n = 40
k = 200 / n = 40
Convergence Rate
Rejection Rate
RMSEA
Intraclass Correlation
0.05
0.25
0.50
0.75
0.95
MSEM
Saturated MSEM
Aggregated CFA

Figure D1. Convergence rates (left column), rejection rate based on the chi-square statistics (middle column), and the average RMSEA (right column) from Simulation Study 2. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines denote a convergence rate of .10 (left column) and a rejection rate of .05, the nominal alpha (middle column).

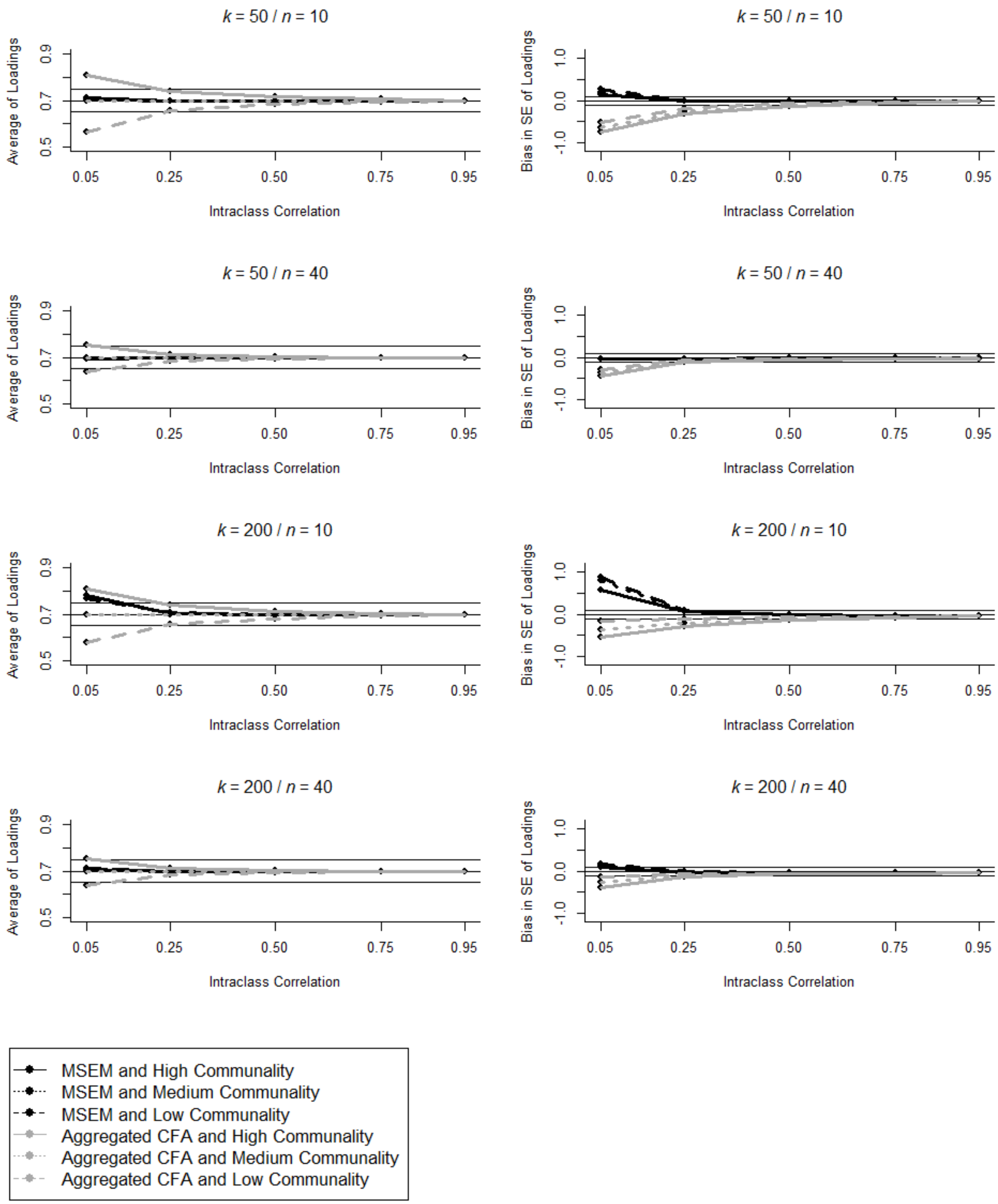


Figure D2. The average standardized factor loadings (left column) and the relative bias in standard errors of standardized factor loadings (right column) in each condition. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines in each plot denote absolute biases of -.05, 0, and .05 to represent the acceptable range of bias for the average standardized factor loadings (left column) and relative biases of -0.1, 0, and 0.1 to represent the acceptable range of bias for the relative bias in the standard errors (right column).

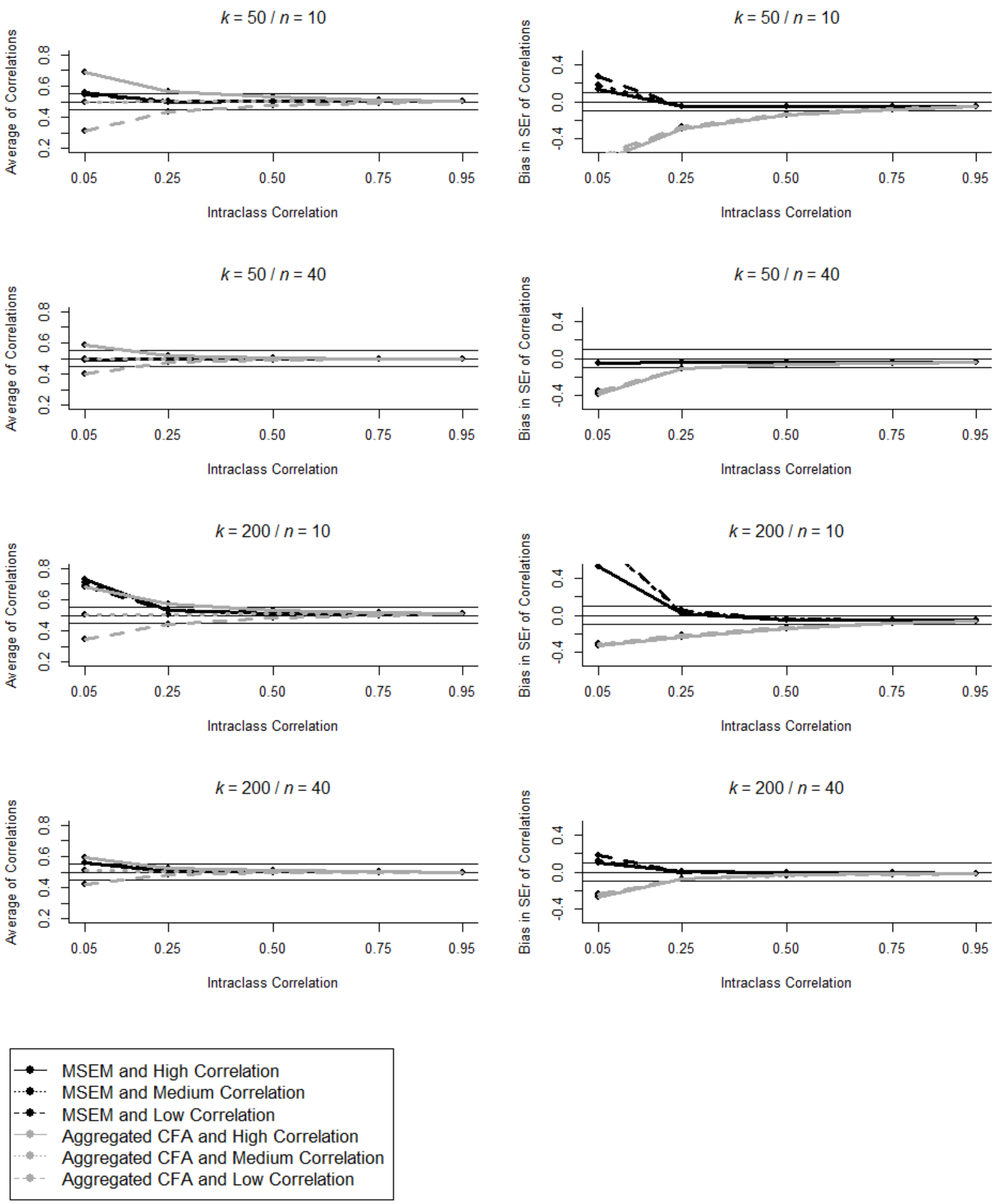


Figure D3. The average factor correlation (left column) and the relative bias in standard errors of factor correlation (right column) in each condition. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines in each plot denote absolute biases of -.05, 0, and .05 to represent the acceptable range of bias for the average factor correlation (left column) and relative biases of -0.1, 0, and 0.1 to represent the acceptable range of bias for the relative bias in the standard errors (right column).

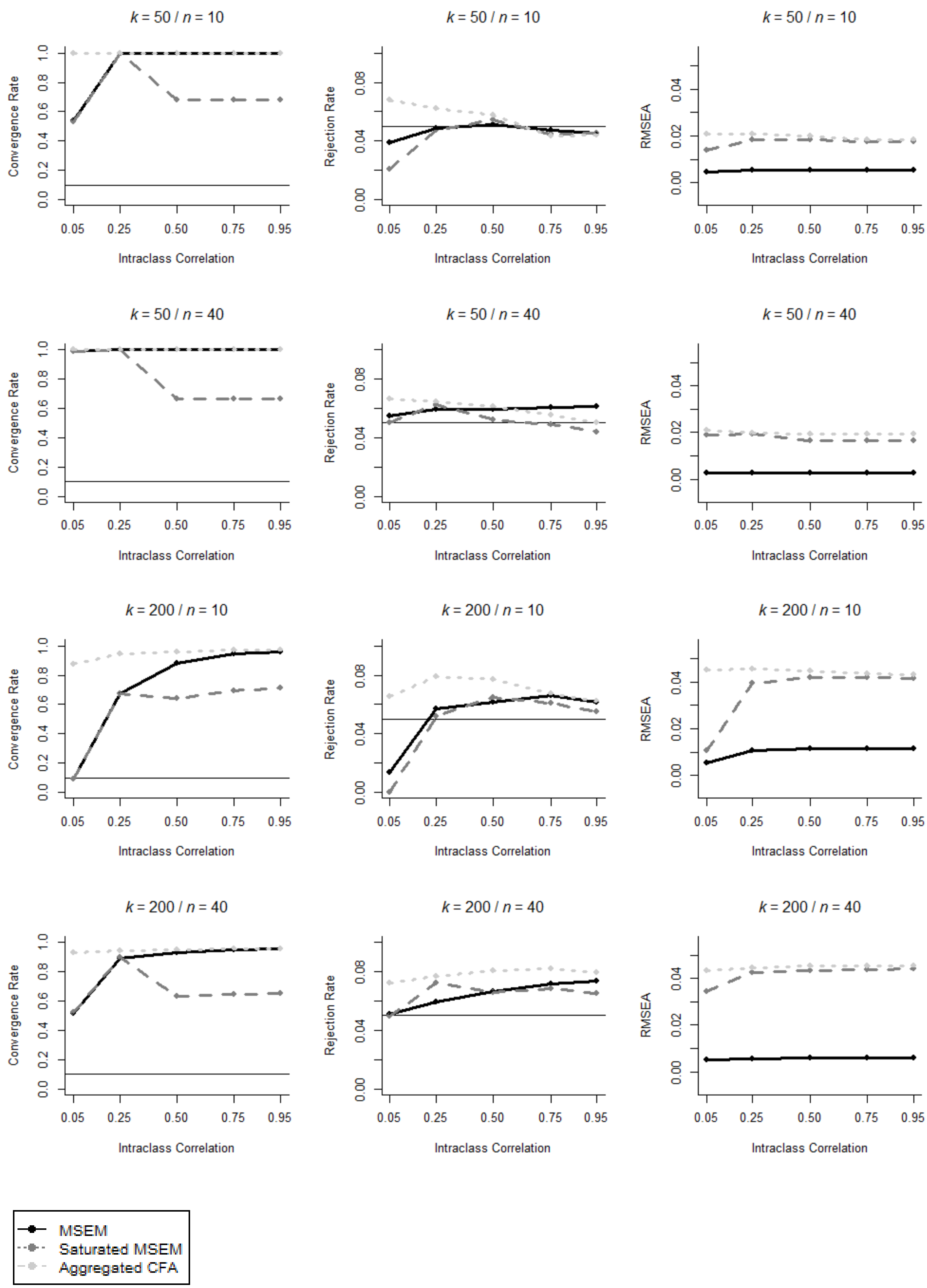

k = 50 / n = 10
Convergence Rate
Rejection Rate
RMSEA
Intraclass Correlation
k = 50 / n = 40
k = 200 / n = 10
k = 200 / n = 40
0.05
0.25
0.50
0.75
0.95
MSEM
Saturated MSEM
Aggregated CFA

Figure D4. Convergence rates (left column), rejection rate based on the chi-square statistics (middle column), and the average RMSEA (right column) of the aggregation simulation when the ICCs are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines denote a convergence rate of .10 (left column) and a rejection rate of .05, the nominal alpha (middle column).

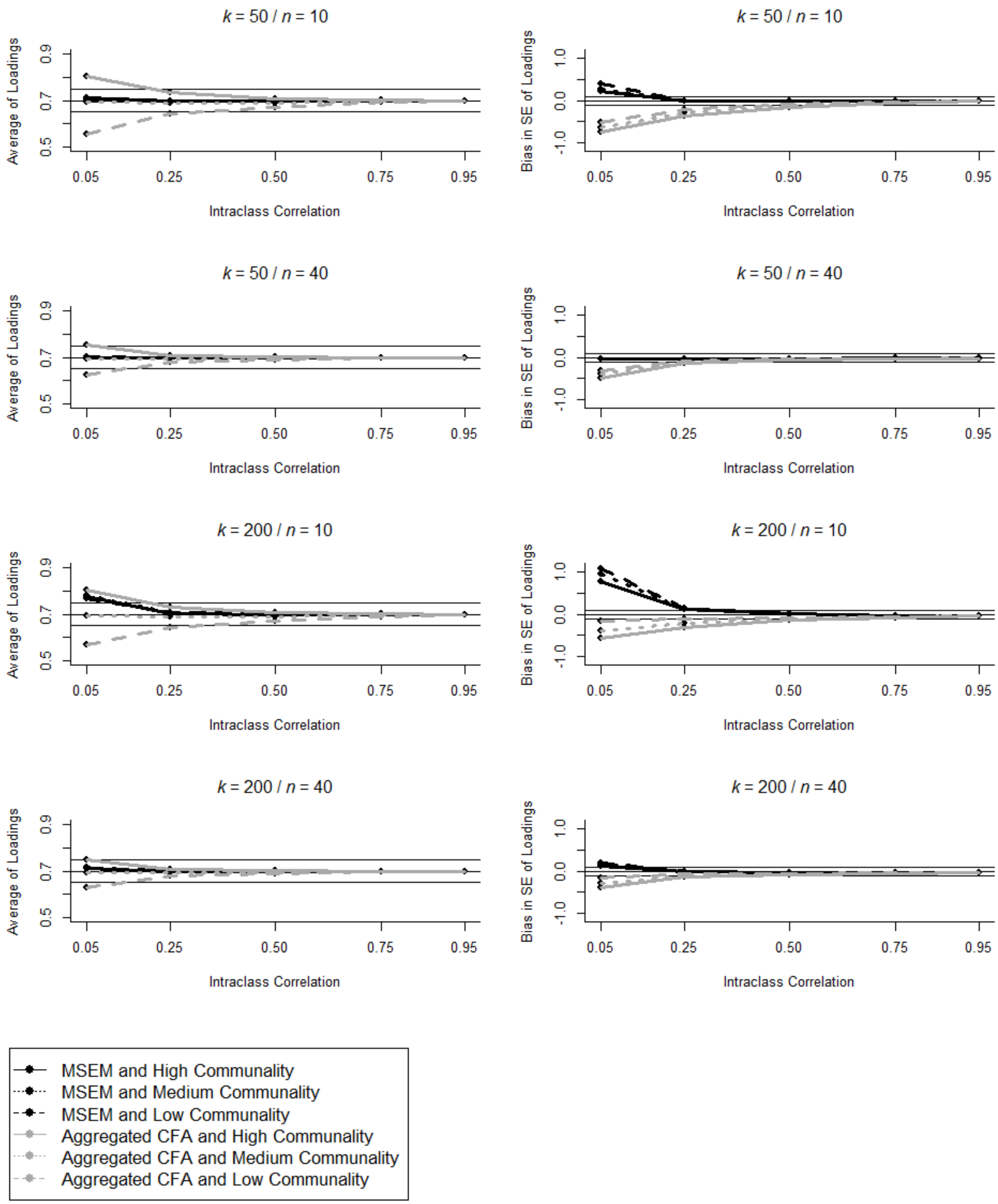


Figure D5. The average standardized factor loadings (left column) and the relative bias in standard errors of standardized factor loadings (right column) in each condition of the aggregation simulation when the ICCs are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines in each plot denote absolute biases of -.05, 0, and .05 to represent the acceptable range of bias for the average standardized factor loadings (left column) and relative biases of -0.1, 0, and 0.1 to represent the acceptable range of bias for the relative bias in the standard errors (right column).

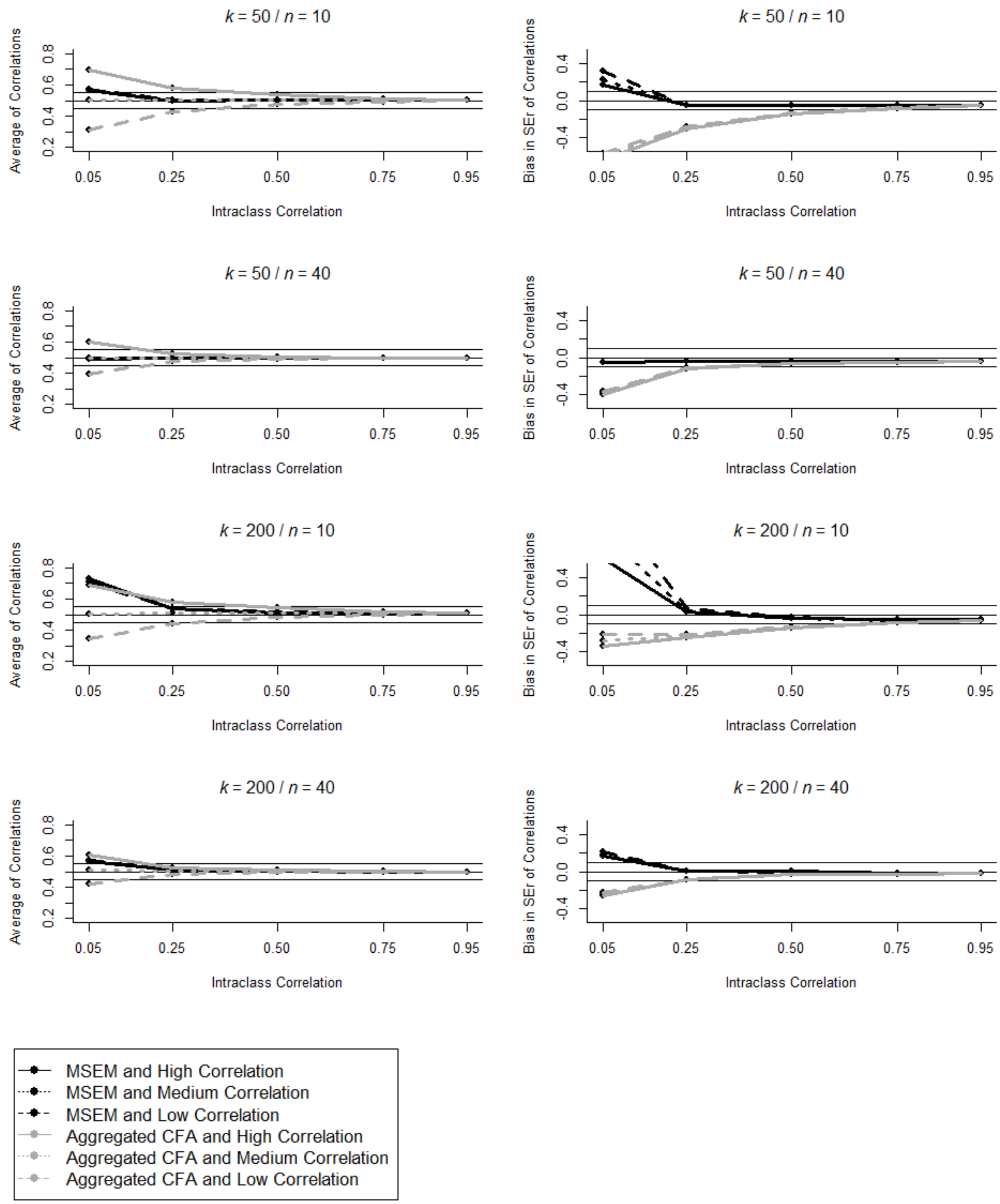
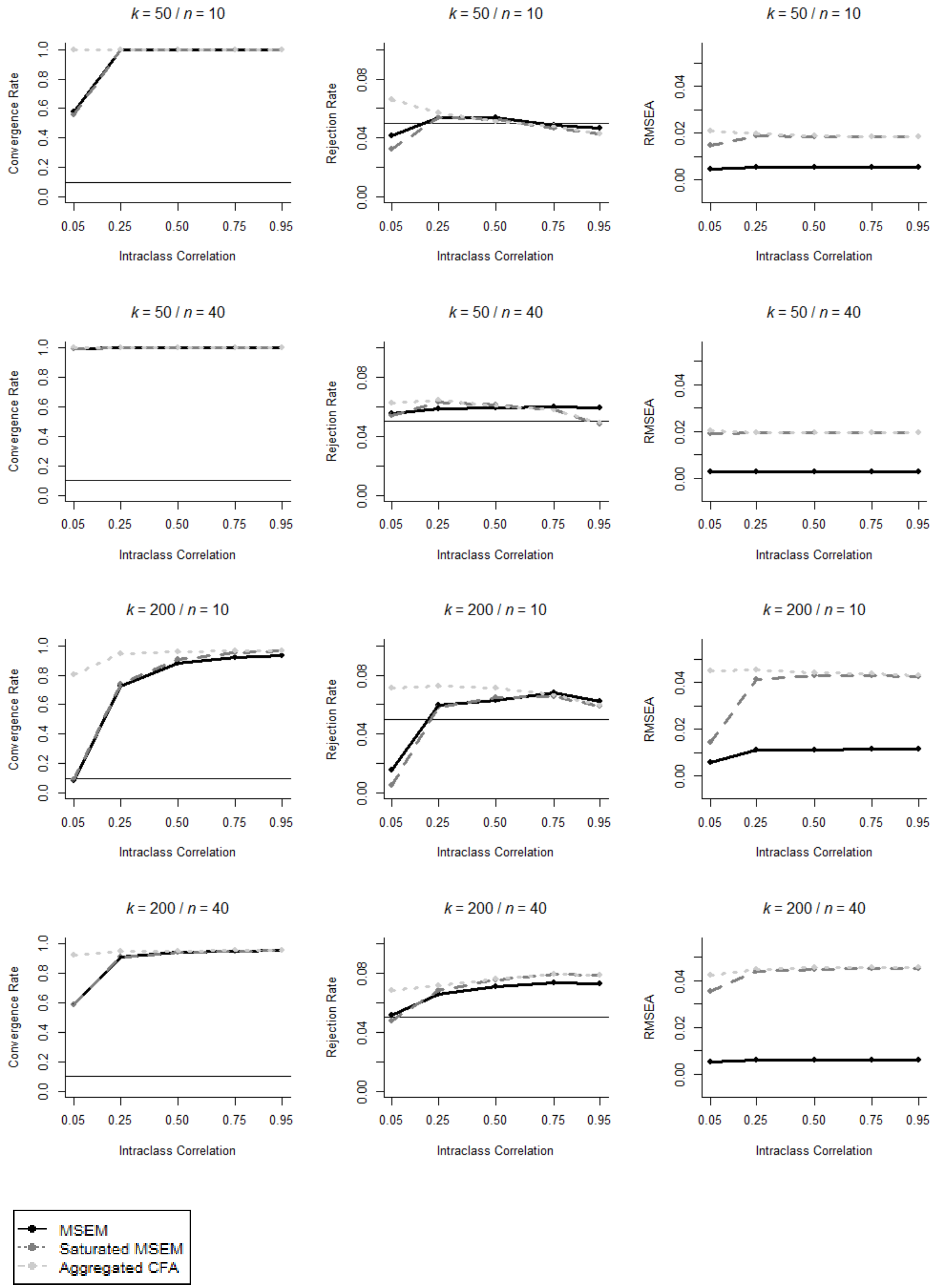


Figure D6. The average factor correlation (left column) and the relative bias in standard errors of factor correlation (right column) in each condition of the aggregation simulation when the ICCs are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines in each plot denote absolute biases of -.05, 0, and .05 to represent the acceptable range of bias for the average factor correlation (left column) and relative biases of -0.1, 0, and 0.1 to represent the acceptable range of bias for the relative bias in the standard errors (right column).



k = 50 / n = 10
k = 50 / n = 40
k = 200 / n = 10
k = 200 / n = 40
Convergence Rate
Rejection Rate
RMSEA
Intraclass Correlation
0.05 0.25 0.50 0.75 0.95
0.0 0.2 0.4 0.6 0.8 1.0
0.00 0.04 0.08
0.00 0.02 0.04
MSEM
Saturated MSEM
Aggregated CFA

Figure D7. Convergence rates (left column), rejection rate based on the chi-square statistics (middle column), and the average RMSEA (right column) of the aggregation simulation when the communalities are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines denote a convergence rate of .10 (left column) and a rejection rate of .05, the nominal alpha (middle column).

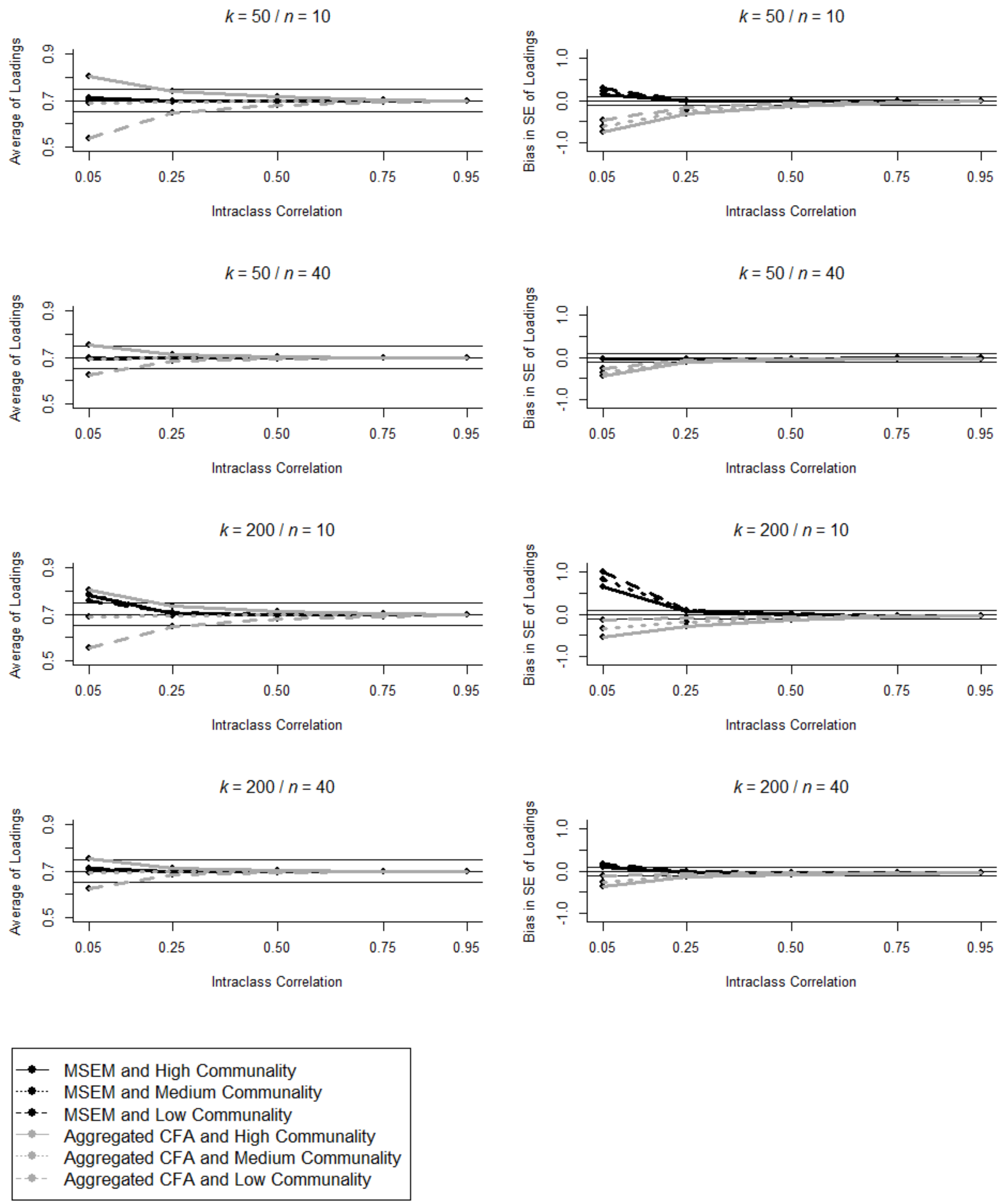


Figure D8. The average standardized factor loadings (left column) and the relative bias in standard errors of standardized factor loadings (right column) in each condition of the aggregation simulation when the communalities are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines in each plot denote absolute biases of -.05, 0, and .05 to represent the acceptable range of bias for the average standardized factor loadings (left column) and relative biases of -0.1, 0, and 0.1 to represent the acceptable range of bias for the relative bias in the standard errors (right column).

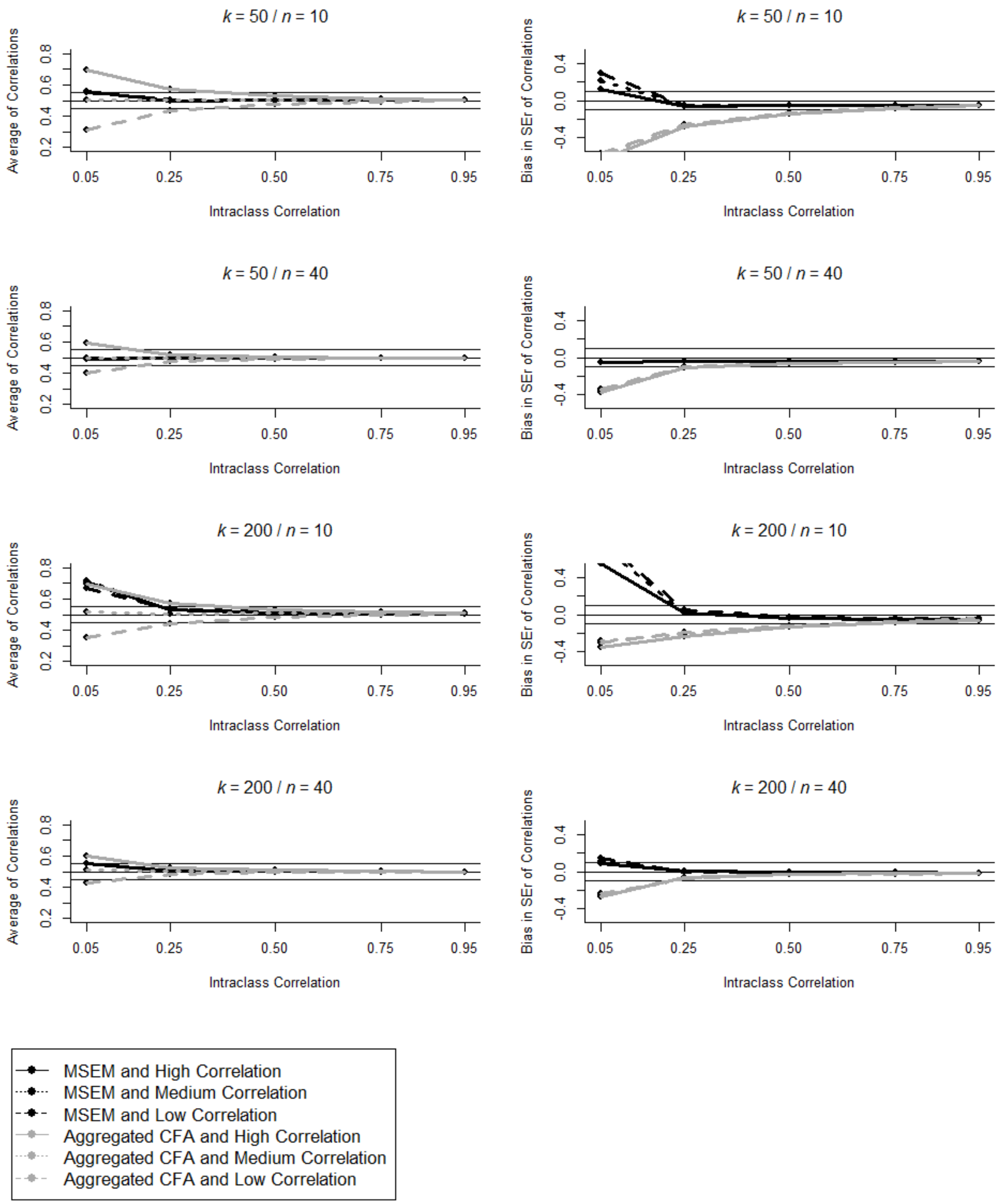


Figure D9. The average factor correlation (left column) and the relative bias in standard errors of factor correlation (right column) in each condition of the aggregation simulation when the communalities are unequal across indicators. *k* is the number of clusters and *n* is the cluster size.

The solid horizontal lines in each plot denote absolute biases of -.05, 0, and .05 to represent the acceptable range of bias for the average factor correlation (left column) and relative biases of -0.1, 0, and 0.1 to represent the acceptable range of bias for the relative bias in the standard errors (right column).

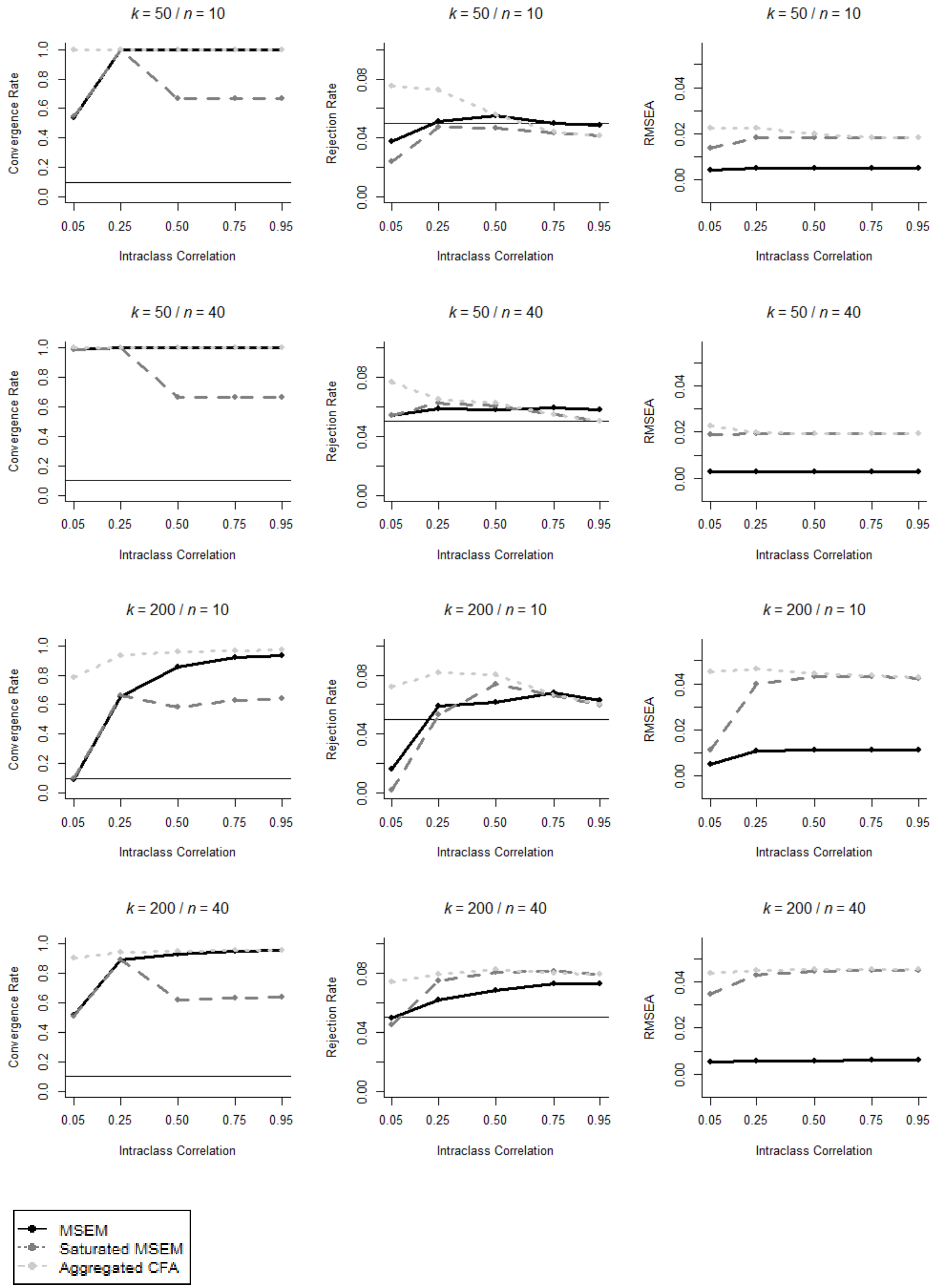

k = 50 / n = 10
k = 50 / n = 40
k = 200 / n = 10
k = 200 / n = 40
Convergence Rate
Rejection Rate
RMSEA
Intraclass Correlation
0.05
0.25
0.50
0.75
0.95
MSEM
Saturated MSEM
Aggregated CFA

Figure D10. Convergence rates (left column), rejection rate based on the chi-square statistics (middle column), and the average RMSEA (right column) of the aggregation simulation when the ICCs and the communalities are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines denote a convergence rate of .10 (left column) and a rejection rate of .05, the nominal alpha (middle column).

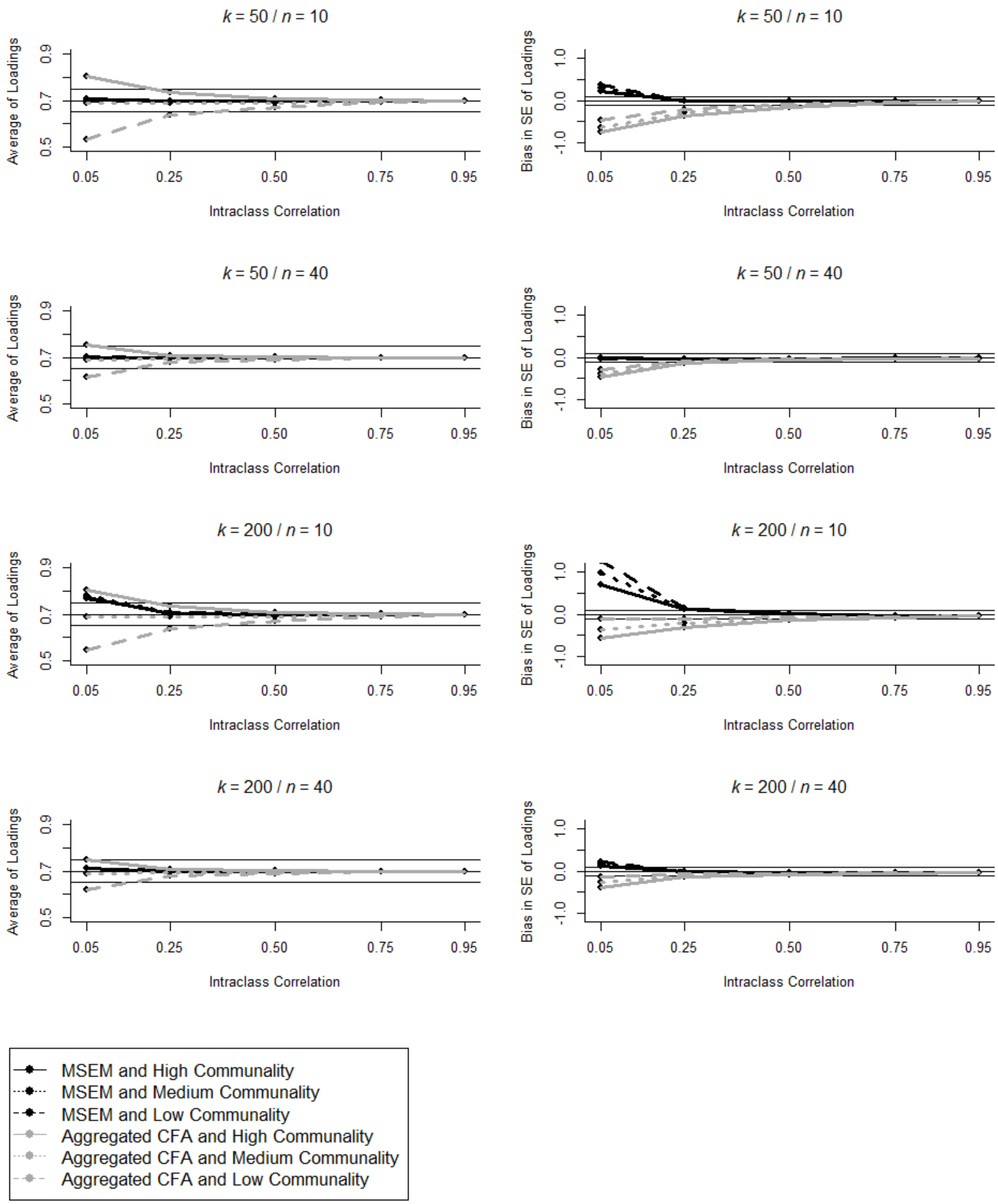


Figure D11. The average standardized factor loadings (left column) and the relative bias in standard errors of standardized factor loadings (right column) in each condition of the aggregation simulation when the ICCs and the communalities are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines in each plot denote absolute biases of -.05, 0, and .05 to represent the acceptable range of bias for the average standardized factor loadings (left column) and relative biases of -0.1, 0, and 0.1 to represent the acceptable range of bias for the relative bias in the standard errors (right column).

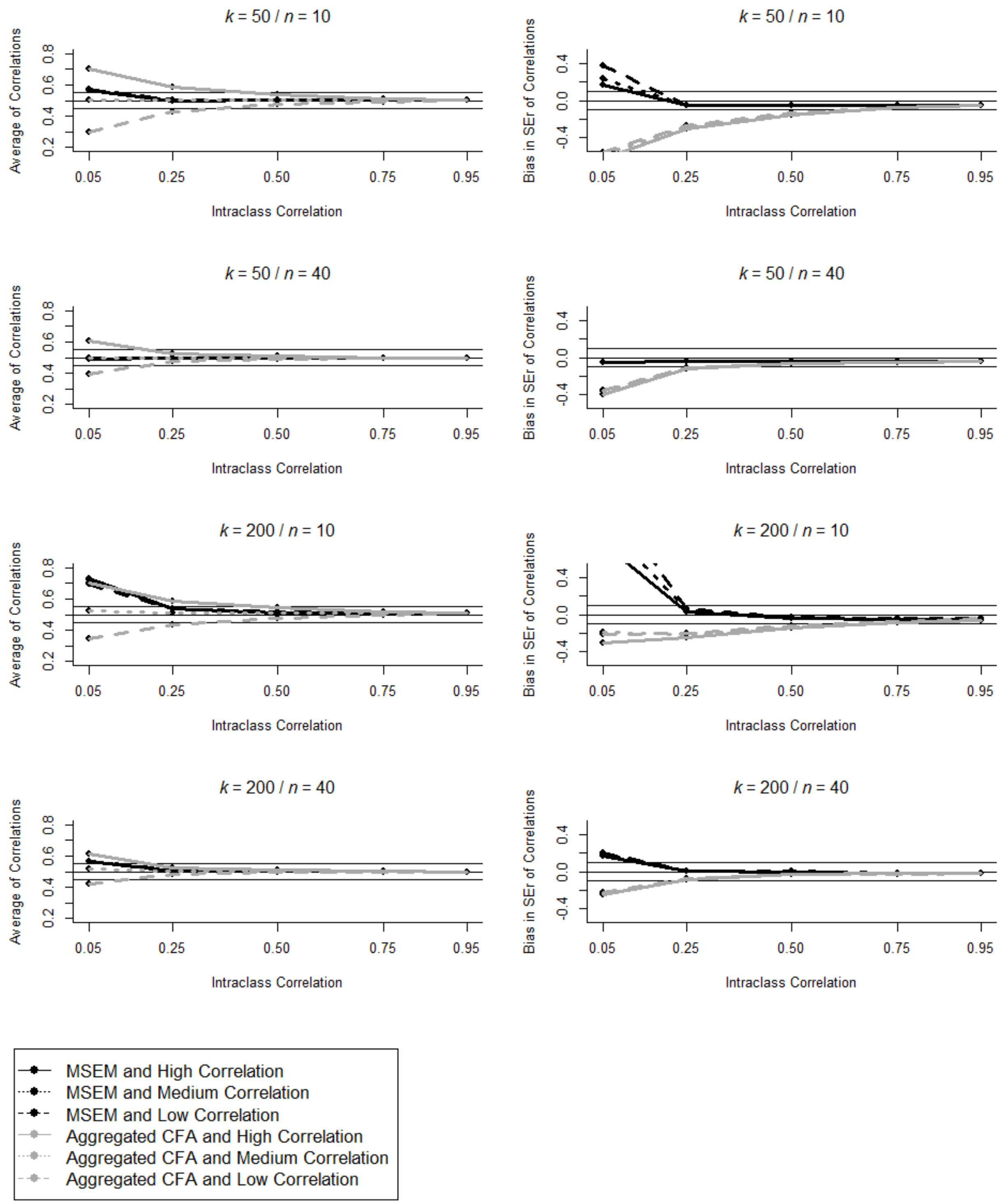


Figure D12. The average factor correlation (left column) and the relative bias in standard errors of factor correlation (right column) in each condition of the aggregation simulation when the ICCs and the communalities are unequal across indicators. *k* is the number of clusters and *n* is the cluster size. The solid horizontal lines in each plot denote absolute biases of -.05, 0, and .05 to represent the acceptable range of bias for the average factor correlation (left column) and relative biases of -0.1, 0, and 0.1 to represent the acceptable range of bias for the relative bias in the standard errors (right column).